



Bayesian Inference and Integrated Data Analysis in Nuclear Fusion



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My Background



1990 Master Technical University, Munich, Germany
1993 PhD Ludwig-Maximilian-University, Munich, Germany
-1996 Postdoc Max-Planck-Institute for Plasma Physics, Garching, Germany
1996- Staff Max-Planck-Institute for Plasma Physics, Garching, Germany
EFDA deputy working group leader; chair of ITPA IDAV SWG; member of ITPA Diagnostics TG

1993 - Data analysis using Bayesian probability theory (~340 Publications)
1999 Initiating Integrated Data Analysis (W7-AS stellarator)
2004 - IDA at ASDEX Upgrade
2014 - IDE kinetic equilibrium reconstruction coupled with current diffusion

ASDEX Upgrade: estimating profiles T_e , n_e , T_i , n_i , Z_{eff} , v_{tor} ; equilibrium reconstruction

Integrated Data Analysis for Nuclear Fusion



Different measurement techniques for the same quantities → redundant and complementary data

Coherent combination of measurements from different diagnostics for plasma control and physics studies

Goal:

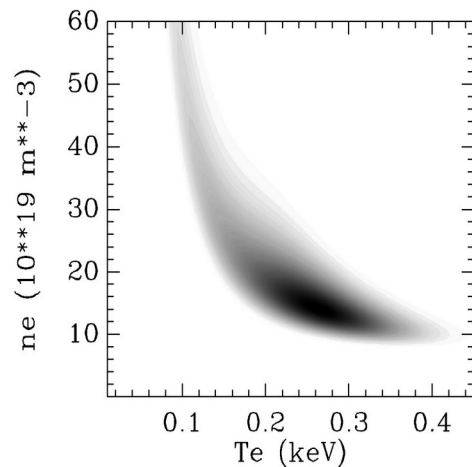
- **replace** combination of **results** from individual diagnostics
- **with** combination of **measured data**
 - one-step analysis of pooled data

Tool:

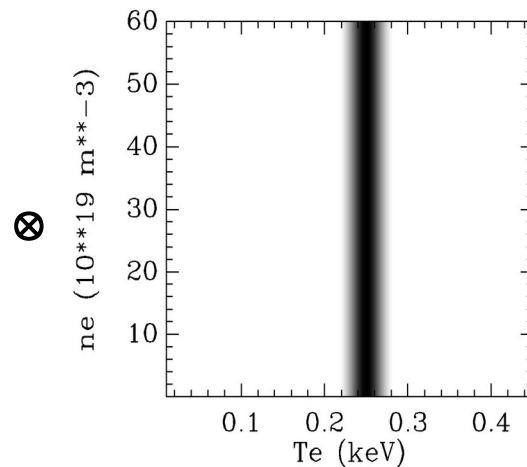
- Bayesian probability theory

Set of diagnostics to be **combined**

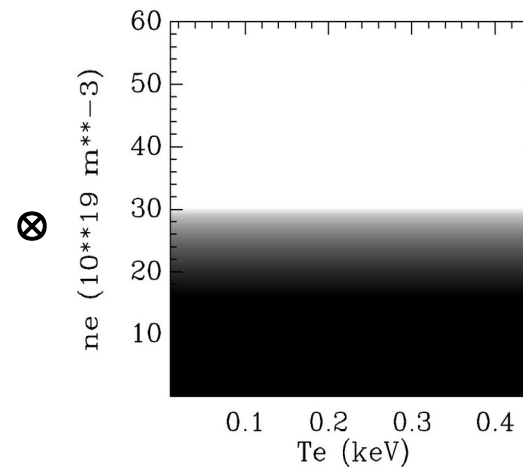
- n_e ... electron density
- T_e ... electron temperature



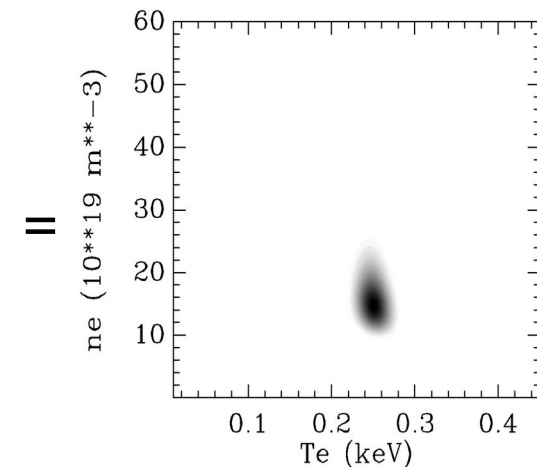
Thomson Scattering



Soft-X-ray



Interferometer
Operation



**Integrated
Result**

Probabilistic framework

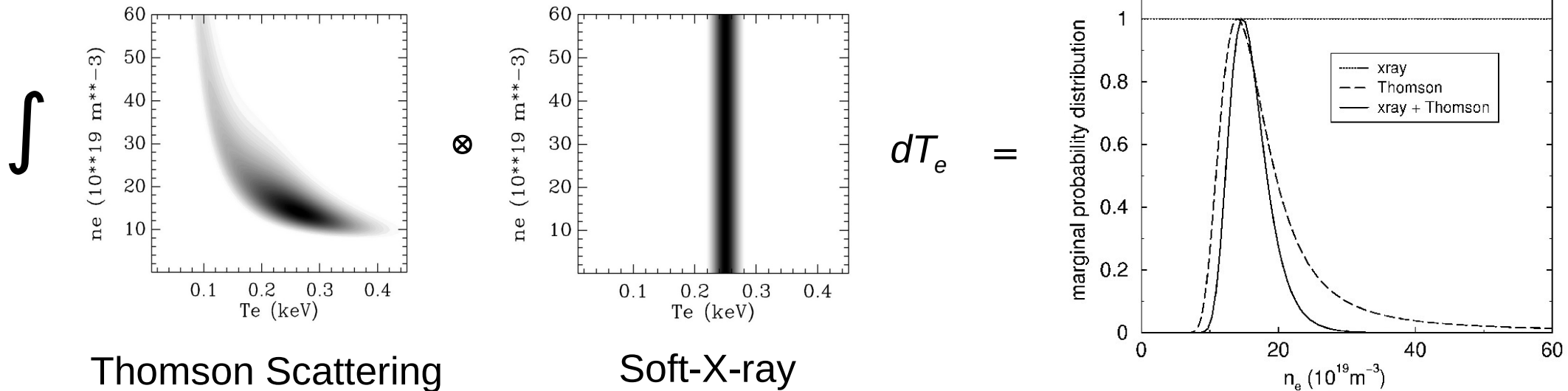
R. Fischer, A. Dinklage, E. Pasch, PPCF 2002, PPCF 2003

Synergistic effect by combined analysis

n_e, T_e : Thomson scattering and soft X-ray

R. Fischer, A. Dinklage, and E. Pasch, Bayesian modelling of fusion diagnostics,
Plasma Phys. Control. Fusion, 45, 1095-1111 (2003)

Using synergism: **Set** of diagnostics to be **combined**



→ synergism by exploiting
full probabilistic correlation structure

Electron density
30% reduced error

- Synergistic effect using probability distributions
- Bayesian concept and recipe
- Conventional vs integrated data analysis
- Examples from ASDEX Upgrade
 - electron density and temperature profiles
 - ion temperature and rotation profiles
 - profile uncertainty
 - magnetic equilibrium reconstruction
- Example from ITER

Why Bayesian Probability Theory?

Scientific inference: prior information + new data $\Rightarrow$ new knowledge

But: prior information and data are uncertain

$\Rightarrow$ new knowledge is uncertain

$\Rightarrow$ propositions (hypotheses) with a ***degree of truth***

$\Rightarrow$ quantification of ***degree of truth*** with probabilities

$\Rightarrow$ ***Bayesian probability theory***

- How to
- handle data uncertainties (statistic, systematic, outliers)
 - handle (uncertain) nuisance parameters
 - exploit prior information
 - combine information / multiple data sets
 - calculate with probability distributions
 - estimate parameters and their uncertainties

- **Thomas Bayes**, 1702-1761, England

minister of the Presbyterian Chapel in Tunbridge Wells

„*Theory of probability in Essay towards solving a problem in the doctrine of chances*“, Philosophical Transactions of the Royal Society of London in 1764

„Inverse problems“

- **Pierre-Simon Laplace**, 1749-1827, France

accepted Bayes conclusions in 1781

Théorie analytique des probabilités, 1812

many applications: mortality, life expectancy, length of marriages, legal matters; errors in observations; the determination of the masses of Jupiter, Saturn and Uranus; triangulation

- **Boole**, 1815-1865, England

„*The Mathematical Theories of Logic and Probabilities*“

- **Andrey Nikolaevich Kolmogorov**, 1903-1987, Russia

„*Analytic methods in probability theory*“



Interpretation of Uncertainties



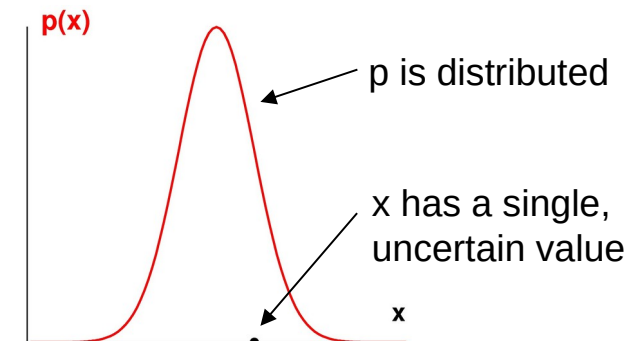
Bayesian: Probability describes uncertainty

- Data:
- statistical (counting)
 - measurement uncertainty (ruler)
 - systematic (mis-alignment, mis-calibration)
 - outliers (unknown or known cause)

- Hypotheses:
- parameter of interest
 - parameter of nuisance
 - number of parameters
 - physical models
 - future data

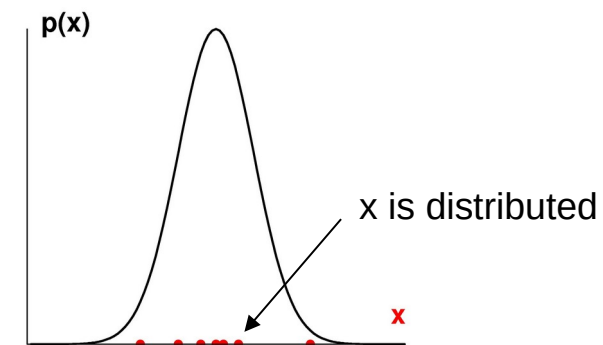
$p(x|I)$ describes how probability (plausibility) is distributed among the possible choices for x in the case at hand (information I)

probability $\neq$ frequency



Frequentist: Probability describes ``randomness``

$p(x|I)$ describes how x is distributed throughout an infinite ensemble:
probability $\equiv$ frequency



Bayesian: Probability describes knowledge conditional on information

Conditional Probability:

$P(A|B)$... Probability of proposition (statement, hypothesis) A given truth of proposition B .
quantification of uncertainty (degree of belief) of A („Bayesian“),
not frequency of outcomes of random variable A („Frequentist“)

Probability Theory Axioms:

• **sum rule** (OR)

$$P(A+B) = P(A) + P(B) - P(A, B)$$

$\Rightarrow$

$$P(B) = \sum_i P(B, A_i) \quad \text{marginalization rule}$$

• **product rule** (AND)

$$P(A, B) = P(A|B) P(B) = P(B|A) P(A)$$

$\Rightarrow$

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)} \quad \text{Bayesian Theorem}$$

$posterior = \frac{likelihood * prior}{evidence}$

Clear instruction how to solve a problem:

If problem is well described than there is only one way to proceed

Example: Sum and Product Rule

Urn with w and b balls
with masses m and M :

α, β	#	$p(\alpha, \beta)$
w, m	100	0.1
w, M	200	0.2
b, m	300	0.3
b, M	400	0.4
	1000	1.0

$$\begin{aligned} P(w) &= P(w, m) + P(w, M) \\ &= 0.1 + 0.2 = 0.3 \end{aligned} \quad \text{sum rule}$$

What is the probability of having a M ball if I drew a w one?

$$\begin{aligned} P(M | w) &= \frac{P(w, M)}{P(w)} && \text{product rule} \\ &= \frac{P(w, M)}{P(w, m) + P(w, M)} && \text{sum rule} \\ &= \frac{0.2}{0.1 + 0.2} = \frac{2}{3} \end{aligned}$$

Bayesian Recipe for IDA: LIB + DCN + ECE + TS

Reasoning about parameter n_e , T_e :

(uncertain) prior information

$$p(n_e, T_e)$$

prior distribution

+ experiment 1: $d_{LiB} = D_{LiB}(n_e, T_e) + \epsilon$; $p(d_{LiB} | n_e, T_e)$

+ experiment 2: $d_{DCN} = D_{DCN}(n_e) + \epsilon$; $p(d_{DCN} | n_e)$

+ experiment 3: $d_{ECE} = D_{ECE}(T_e) + \epsilon$; $p(d_{ECE} | T_e)$

+ experiment 4: $d_{TS} = D_{TS}(n_e, T_e) + \epsilon$; $p(d_{TS} | n_e, T_e)$

likelihood
distributions

+ Bayes theorem $p(n_e, T_e | d_{TS}, d_{ECE}, d_{LiB}, d_{DCN}) \propto p(d_{TS} | n_e, T_e) \times$
 $p(d_{ECE} | T_e) \times$
 $p(d_{LiB} | n_e, T_e) \times$
 $p(d_{DCN} | n_e) \times$
 $p(n_e, T_e)$

posterior
distribution

+ additional uncertain (nuisance) parameter $\rightarrow$ marginalization

$$p(n, T | d) = \int p(n, T | d, \alpha) p(\alpha) d\alpha$$

generalization of Gaussian error propagation laws

Likelihood probability distribution

describes the uncertainty/statistics of **data**:

forward modeled (synthetic) data:

noise (measurement uncertainty):

$$d_i = D(\mathbf{T}, x_i, \alpha) + \epsilon_i$$

$$D(\mathbf{T}, x_i, \alpha)$$

$$\epsilon_i$$

likelihood:

$$p(d_i | \mathbf{T}) = p(\epsilon_i = d_i - D(\mathbf{T}, x_i, \alpha))$$

- Gaussian: for independent, normally distributed measurement errors with uncertainty σ

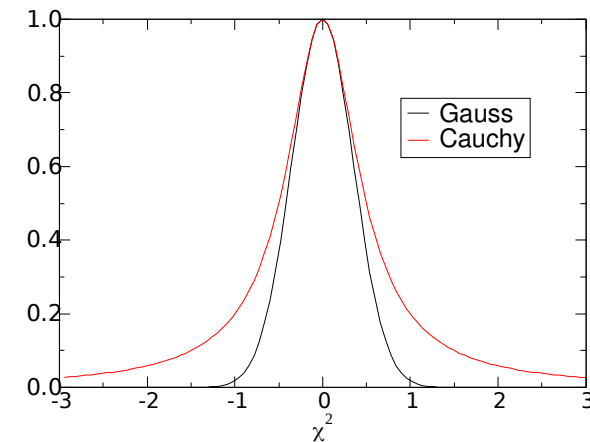
$$\langle \epsilon \rangle = 0$$

$$\langle \epsilon^2 \rangle = \sigma^2$$

$$p(\vec{d} | \mathbf{T}, \vec{\sigma}) = \frac{1}{\prod_i \sqrt{2\pi\sigma_i^2}} \exp\left\{-\frac{\chi^2}{2}\right\}$$

with

$$\chi^2 = \sum_i^N \frac{[d_i - D_i(\mathbf{T})]^2}{\sigma_i^2}$$



- Cauchy, Student's-t: robust estimation (outliers, data failures)

→ workhorse in fusion data analysis

$$p(\vec{d} | \mathbf{T}, \vec{\sigma}) \propto \prod_i^N \{a + \chi_i^2/2\}^{-(a+0.5)}$$

- Poisson: Counting experiments $p(\vec{d} | D(\lambda, \vec{x})) = \prod_i^N \frac{D_i^{d_i}}{d_i!} \exp(-D_i)$

Quantification of relevant additional information that is available **independent** of the **data**

Data Analysis - A Bayesian Tutorial, D. S. Sivia, Clarendon (1996)

➤ non-physical:

- smoothness: Tikhonov, MaxEnt, min-Fisher information, ...
- number of parameters, e.g. for profiles
- correlation lengths, e.g. Gaussian process regression (GPR)

R. Fischer et al., Integrated Data Analysis and Validation, Chap. 10, NF, to be published
arXiv:2411.09270 [physics.plasm-ph]

➤ physical:

- positivity constraints, e.g. n_e , T_e , n_i , T_i ; boundaries $Z_{\text{eff}} \geq 1$; monotonicity, moments
- calibration measurements + **uncertainties**
- data bases, atomic data including their **uncertainties**
- predictive modeling
 - parameters (T_e , n_e , T_i , v_{tor} , ...) are physically correlated
 - example: transport codes providing profile (logarithmic) gradients or upper limits of gradients
ASTRA-TGLF kinetic modeling

(M. Bergmann, Nuclear Fusion 64 (2024) 056024)

To obtain most reliable results

- 1) exploit all (uncertain) information you have
- 2) quantify it with probability distributions
- 3) and multiply it to the likelihood

Bayesian Estimates

Posterior distribution: $p(\theta|d) = \frac{p(d|\theta) \times p(\theta)}{p(d)}$

Best estimates:
 $\max_{\theta} p(\theta|d) \rightarrow \hat{\theta}$
 $\text{mean}_{\theta} p(\theta|d) \rightarrow \langle \theta \rangle$

Uncertainties:

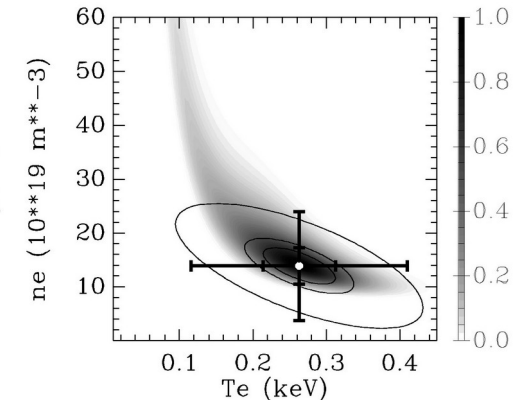
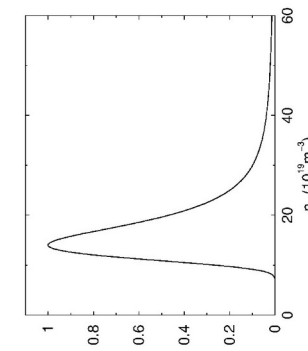
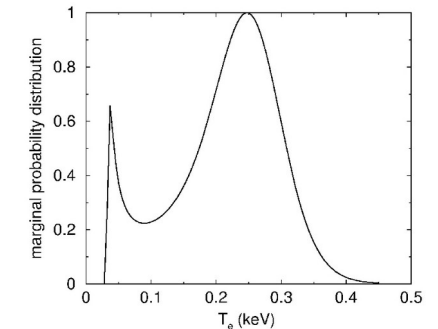
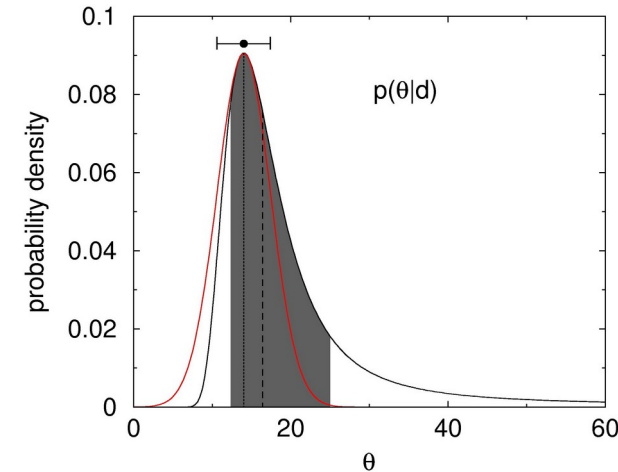
Laplace approx. $\text{var}_{\theta} p(\theta|d)|_{\hat{\theta}} \rightarrow \sigma_{\theta}^2$

distribution variance $\text{var}_{\theta} p(\theta|d) \rightarrow \Delta \theta^2$

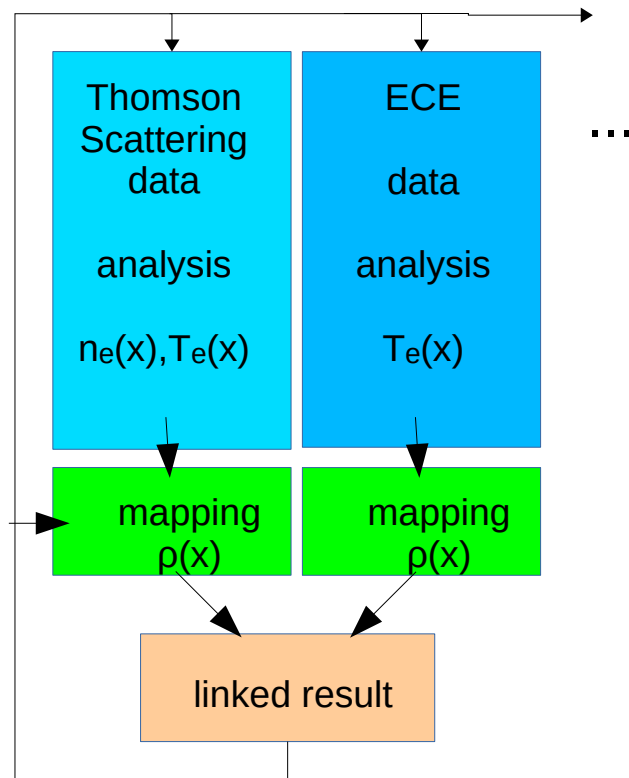
credibility (confidence) regions

68.3% , 95.4% , 99.73% -intervals

(50±34)% percentile

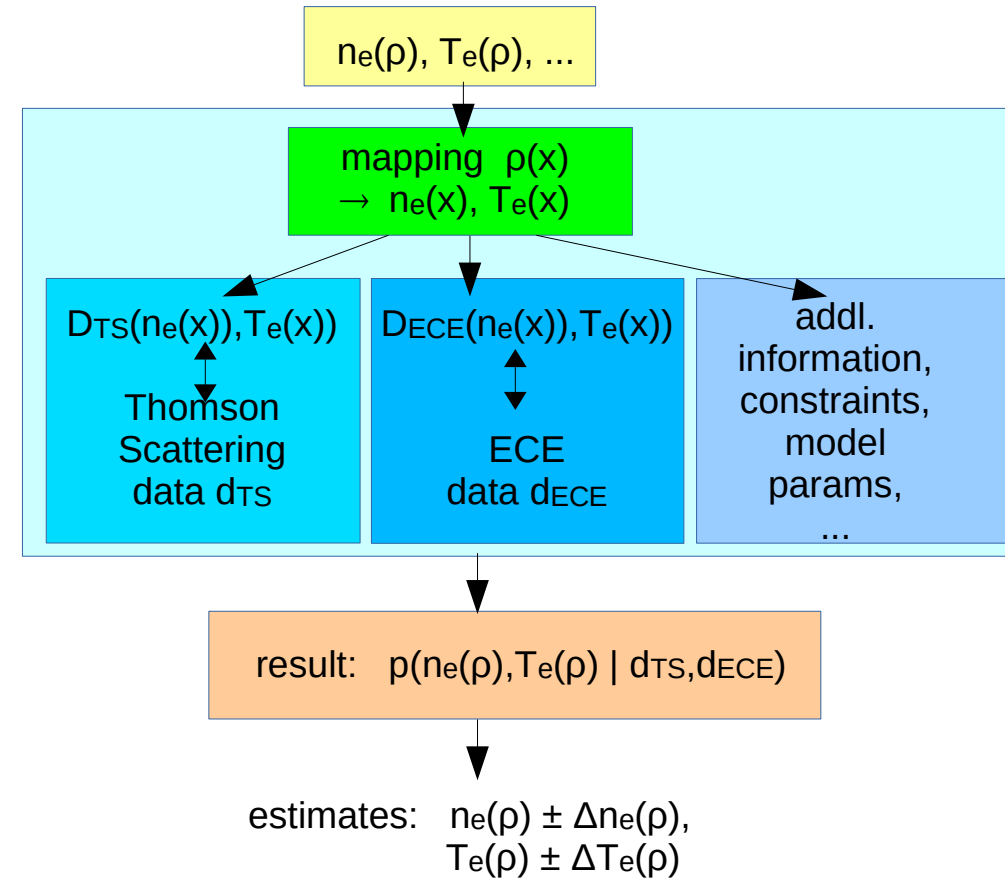


conventional



Parametric entanglements

IDA (Bayesian probability theory)



Conventional vs. Integrated Data Analysis (cont.)

Drawbacks of conventional data analysis: iterative

- (self-)consistent results? (cumbersome; do they exist?)
- difficult to be automated (huge amount of data from steady state devices: ITER, ...)
- information propagation? (Single estimates as input for analysis of other diagnostics?)
- error propagation? (frequently neglected: underestimation of the uncertainty)
- data and result validation? (How to deal with inconsistencies?)
- often backward inversion techniques (noise fitting? numerical stability?)
- result: estimates and error bars (sufficient? non-linear dependencies?)

Probabilistic combination of different diagnostics (IDA)

- ✓ uses only forward modeling (complete set of parameters → modeling of measured data)
- ✓ additional physical information easily to be integrated
- ✓ systematic effects (inconsistency) → describe with (nuisance) parameters
- ✓ unified error interpretation → Bayesian Probability Theory
- ✓ result: probability distribution of parameters of interest incl. all dependencies

IDA offers a unified way
of combining data (information) from various experiments (sources)
to obtain improved results

Axial Symmetric Divertor EXperiment

ASDEX 1980-1990

ASDEX Upgrade 1990-

Garching, near Munich, Germany

mid-size tokamak (similar to DIII-D)

- major plasma radius 1.65 m
- minor plasma radius 0.5 m
- plasma volume 14 m³
- max magnetic field 3.1 T
- plasma current 0.4 MA - 1.6 MA
- pulse duration 10 s
- plasma types: deuterium, hydrogen, helium
- plasma heating: 35 MW
 - ohmic 1 MW
 - neutral beam injection 20 MW
 - ion cyclotron 6 MW
 - electron cyclotron 8 MW for 10 s



Highlights:

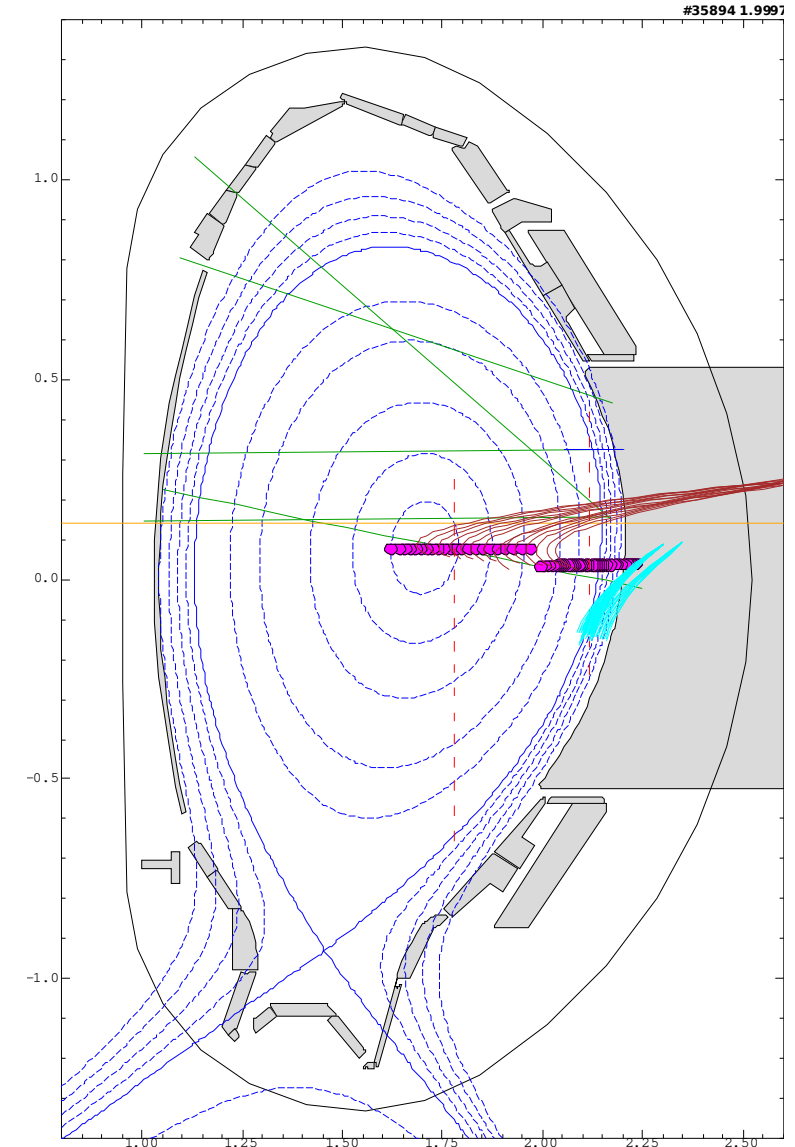
- invention of the divertor
- discovery and description of the H-mode
- metallic wall

Application: ASDEX Upgrade

multi-diagnostic profile reconstruction: n_e , T_e

- Lithium beam impact excitation spectroscopy (LIB)
collisional radiative model $\rightarrow n_e(T_e)$
 $\rightarrow n_e$
 - Interferometry measurements (DCN) $\rightarrow n_e$
 - Electron cyclotron emission (ECE)
ECRad: Electron cyclotron radiation transport $\rightarrow T_e(n_e)$
 - Thomson scattering (TS) $\rightarrow n_e, T_e$
 - Reflectometry $\rightarrow n_e$
 - Beam emission spectroscopy $\rightarrow n_e(Z_{eff})$
 - Thermal Helium beam spectroscopy $\rightarrow n_e, T_e$
-
- Equilibrium reconstructions for diagnostics mapping
(IDE: kinetic Grad-Shafranov solution coupled with current diffusion)

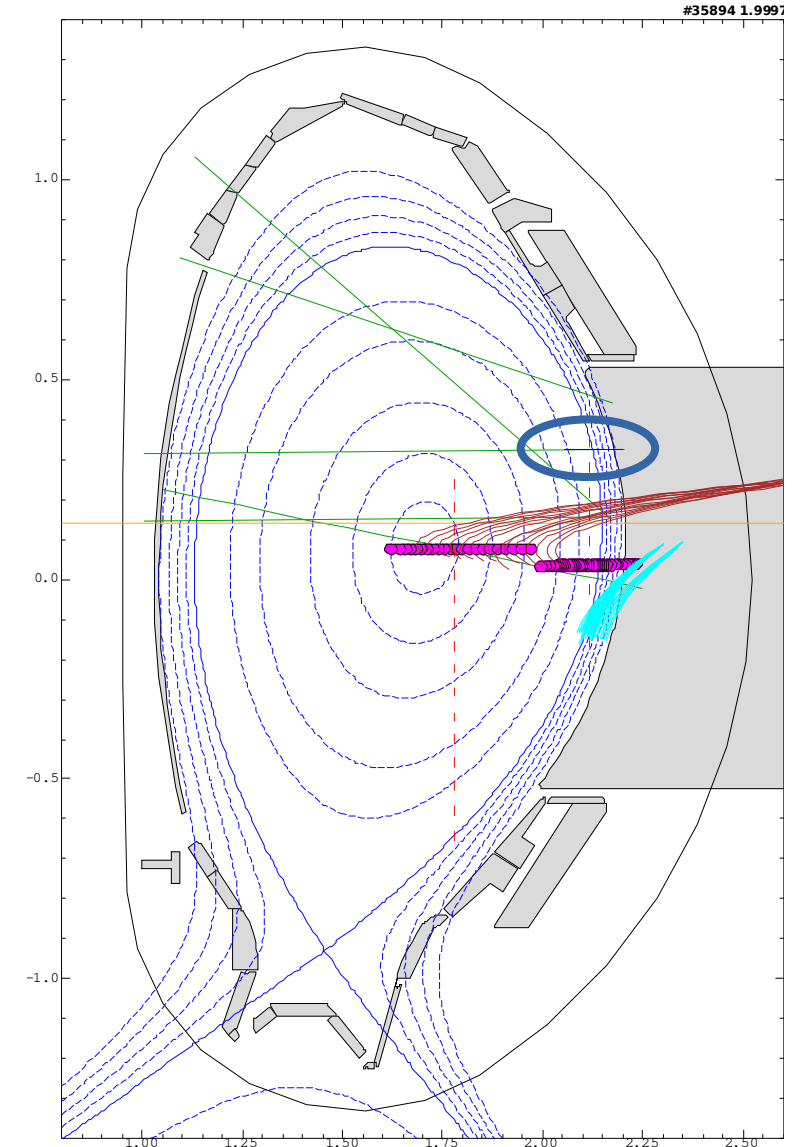
A lot of dependencies and uncertainties:
We need a probabilistic approach!



Application: ASDEX Upgrade

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collisional radiative model $\rightarrow n_e(T_e)$
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- Beam emission spectroscopy $\rightarrow n_e(Z_{eff})$
- Thermal Helium beam spectroscopy $\rightarrow n_e, T_e$



Lithium beam impact excitation spectroscopy

LiI radiation from neutral Lithium

$E=30-80$ keV

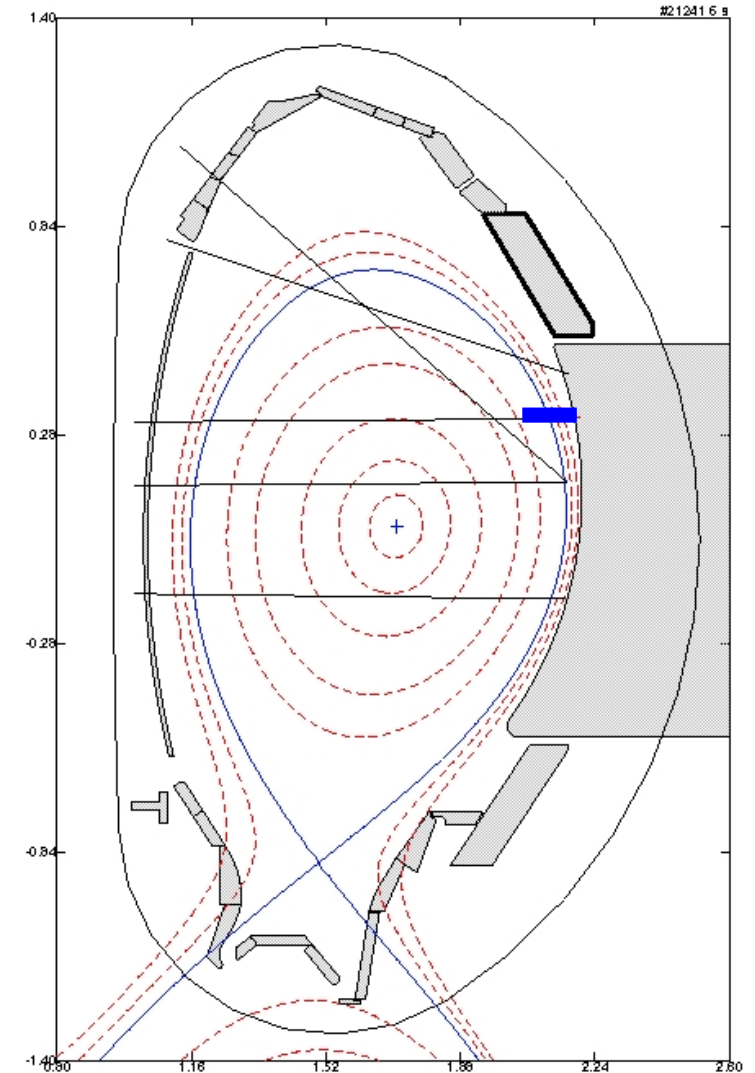
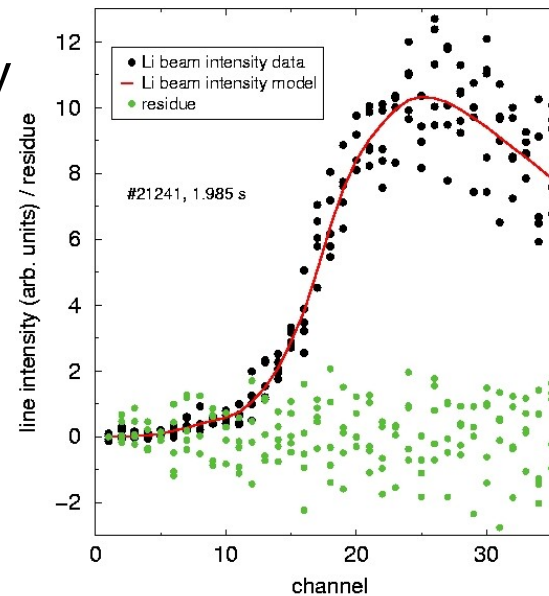
$\text{Li}(2p) \rightarrow \text{Li}(2s), \lambda = 670.8$ nm

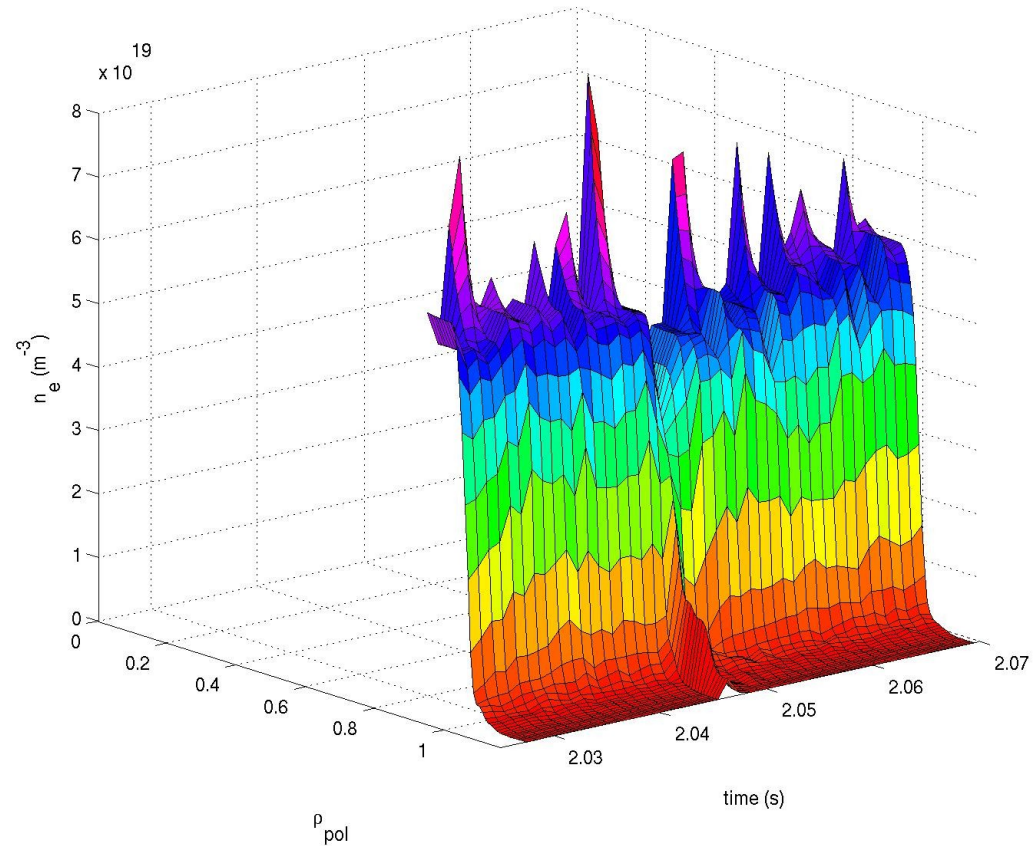
Solve a collisional radiative model

System of coupled linear differential equations:

$$\frac{dN_i(x)}{dx} = \sum_{j=1}^{N_{\text{Li}}} \{ n_e(x) a_{ij}(T_e(x)) + b_{ij} \} N_j(x) ; \quad N_i(x=0) = \delta_{1i}$$

solved for a given profile $n_e(x)$ to obtain
occupation density Li(2p): $N_2(x|n_e)$





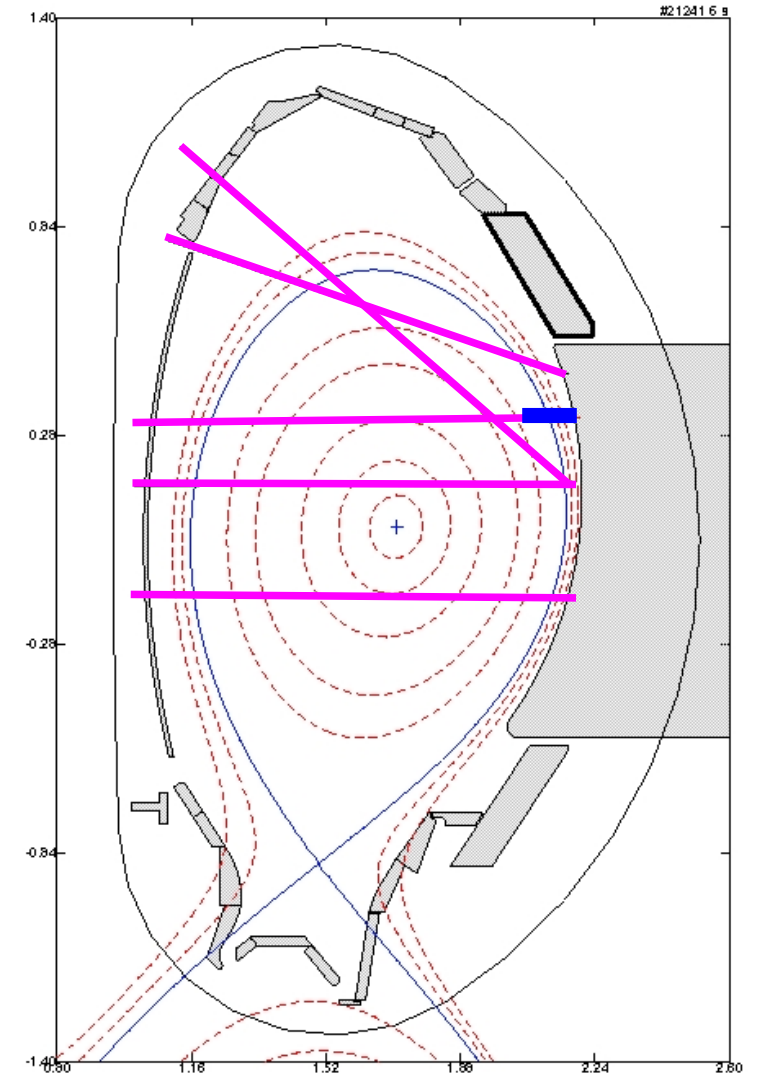
LIB: Lithium beam only
→ edge density profile

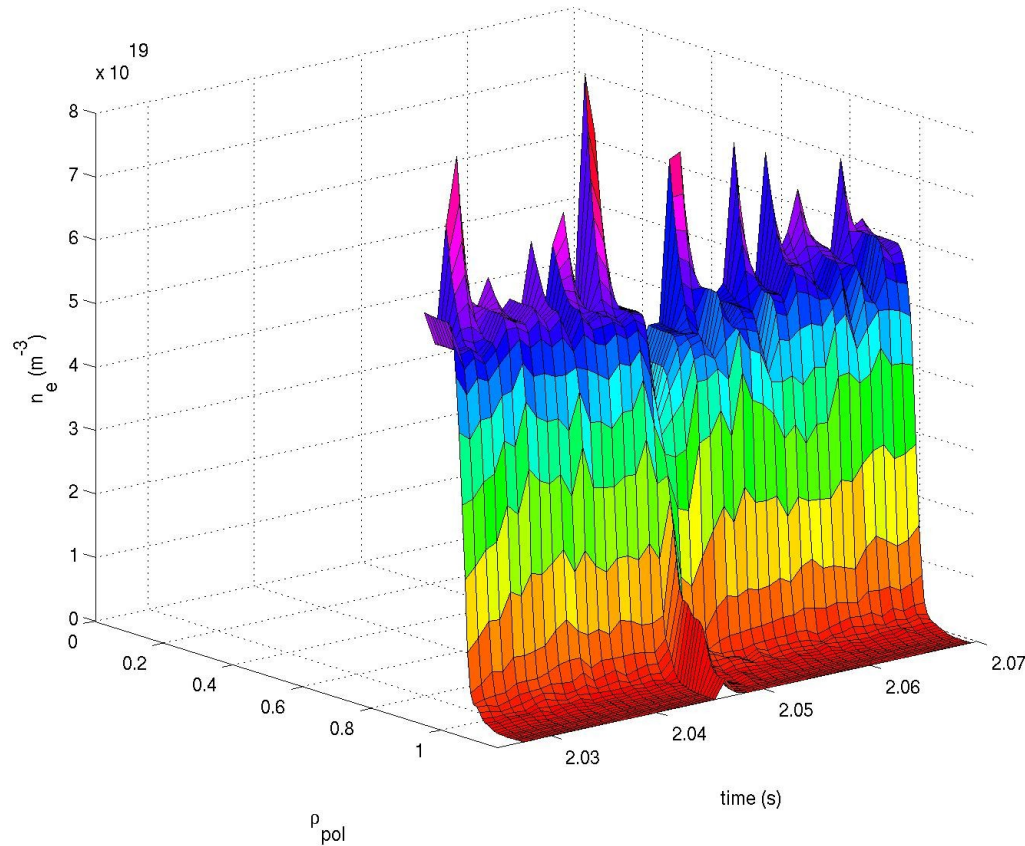
Interferometry with DCN laser:

- Intensity proportional to line integrated electron density

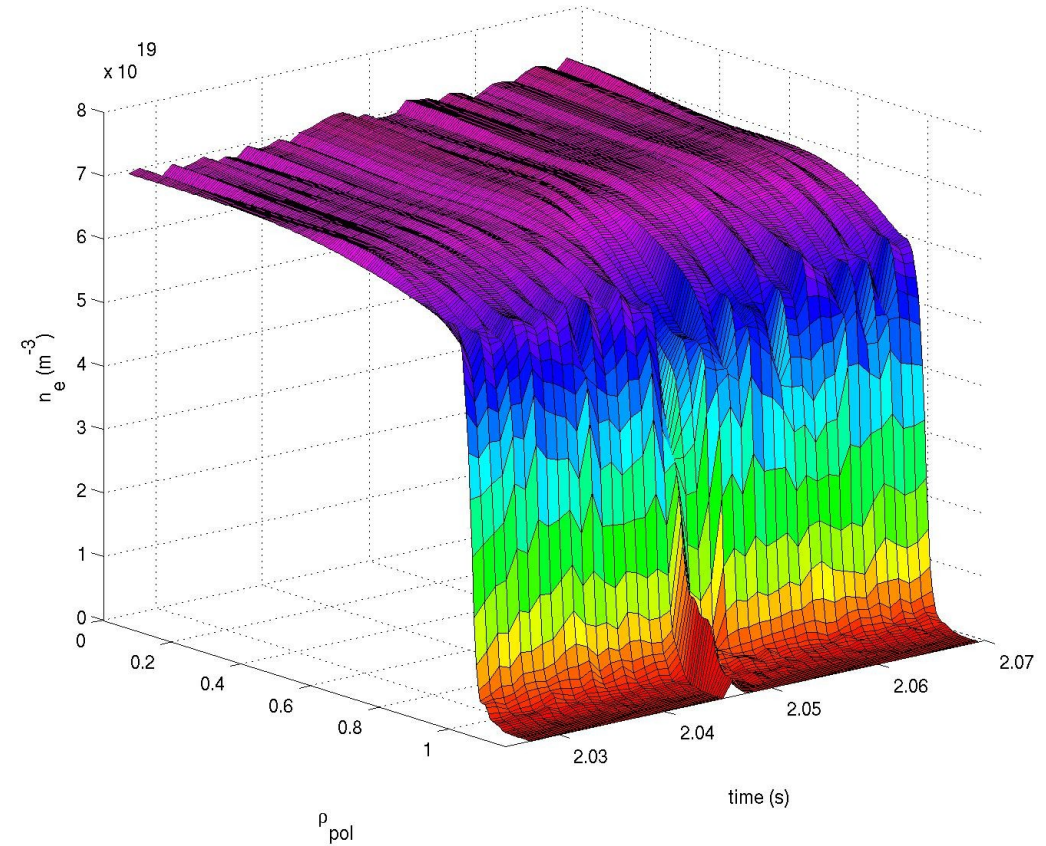
$$d \propto \int n_e dl$$

- poor spatial resolution
- reliable absolute calibration

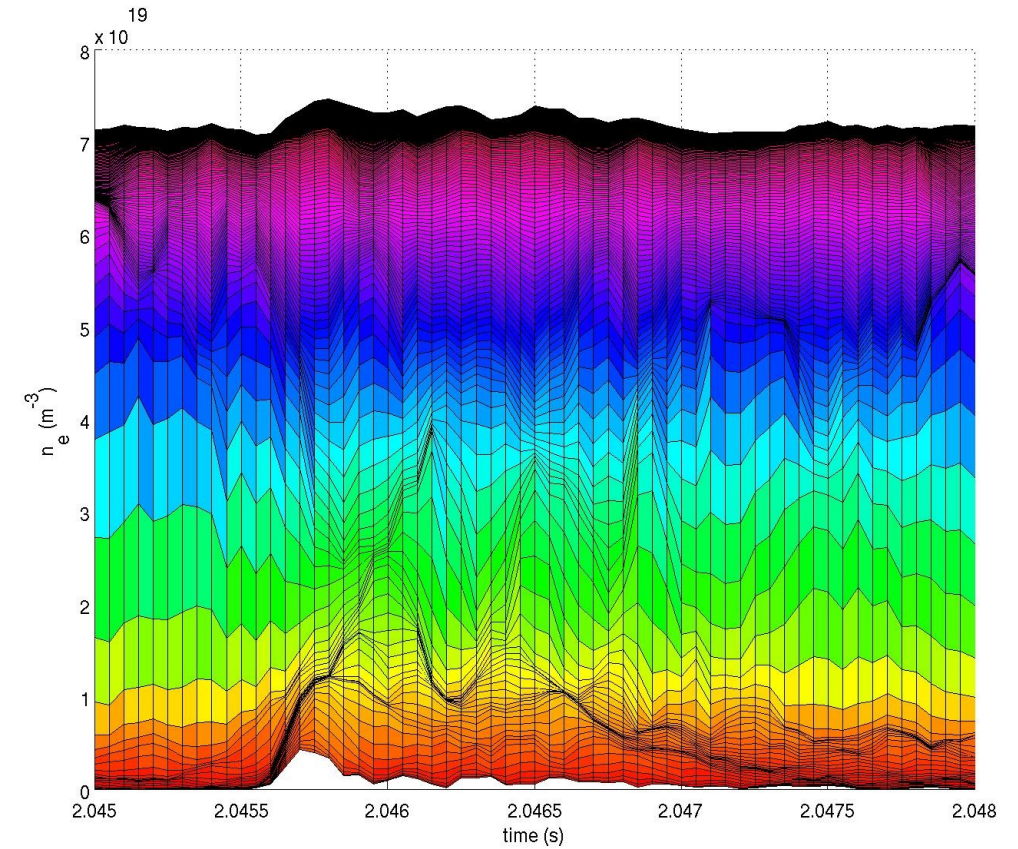
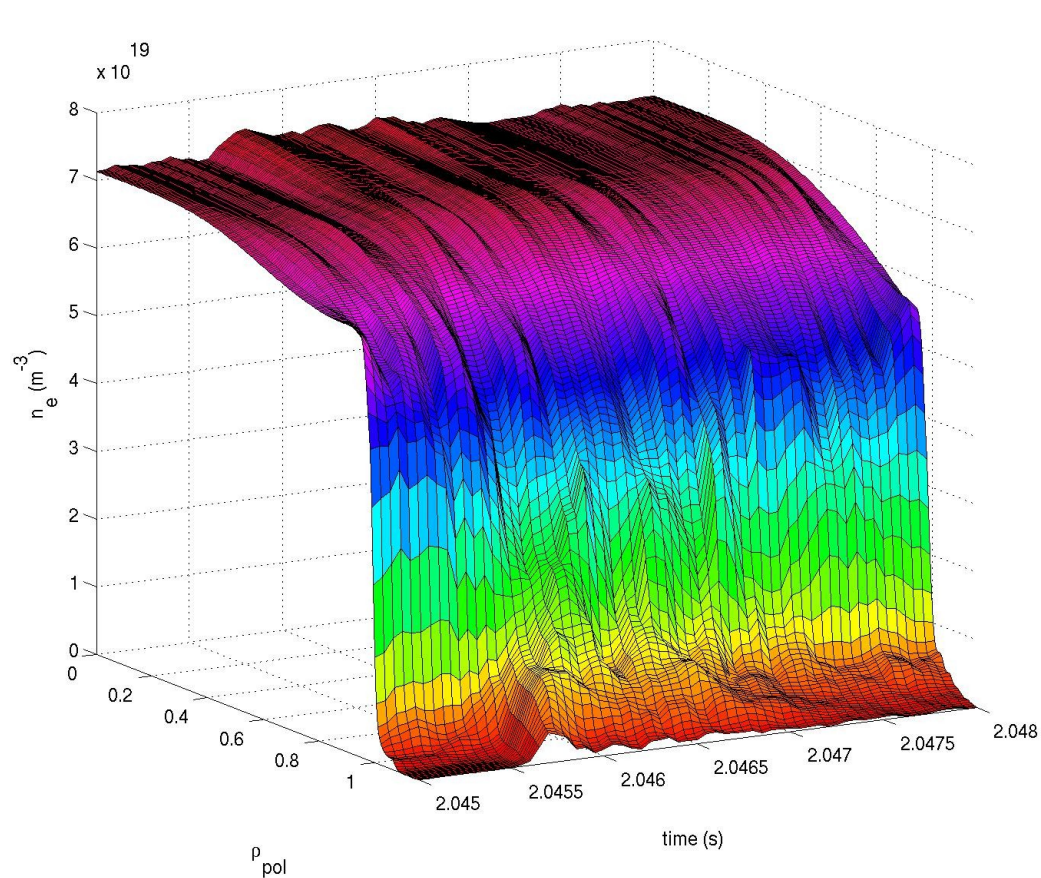




LIB: Lithium beam only
→ edge density profile



IDA: Lithium beam + DCN Interferometry
→ full density profile

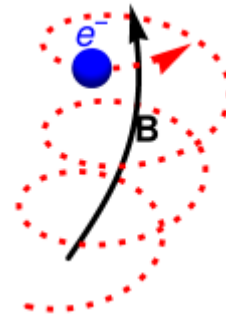


- temporal resolution 5 μ s (5 ms with old LIB inversion technique)
- triggered installation of new optical head $\rightarrow$ fluctuation measurements

Electron cyclotron emission:

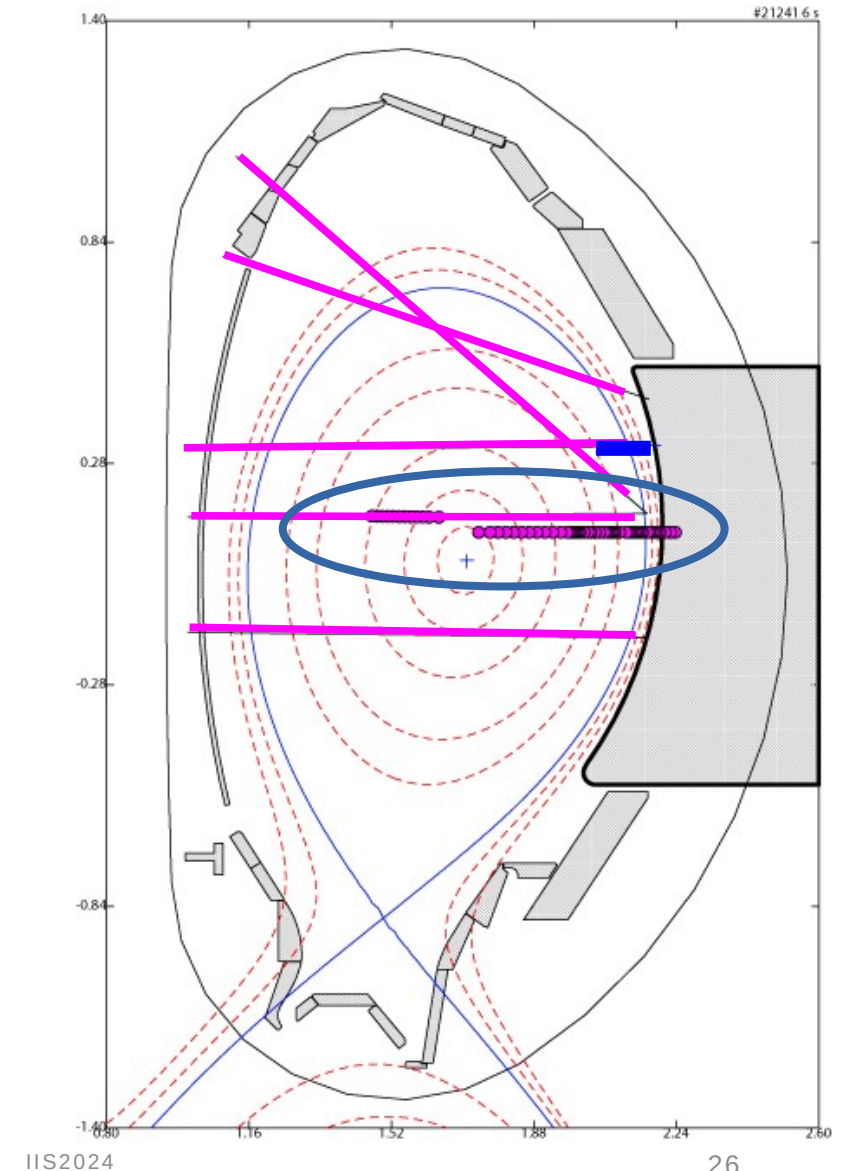
$$\omega_c = \frac{e B(R)}{m_e}$$

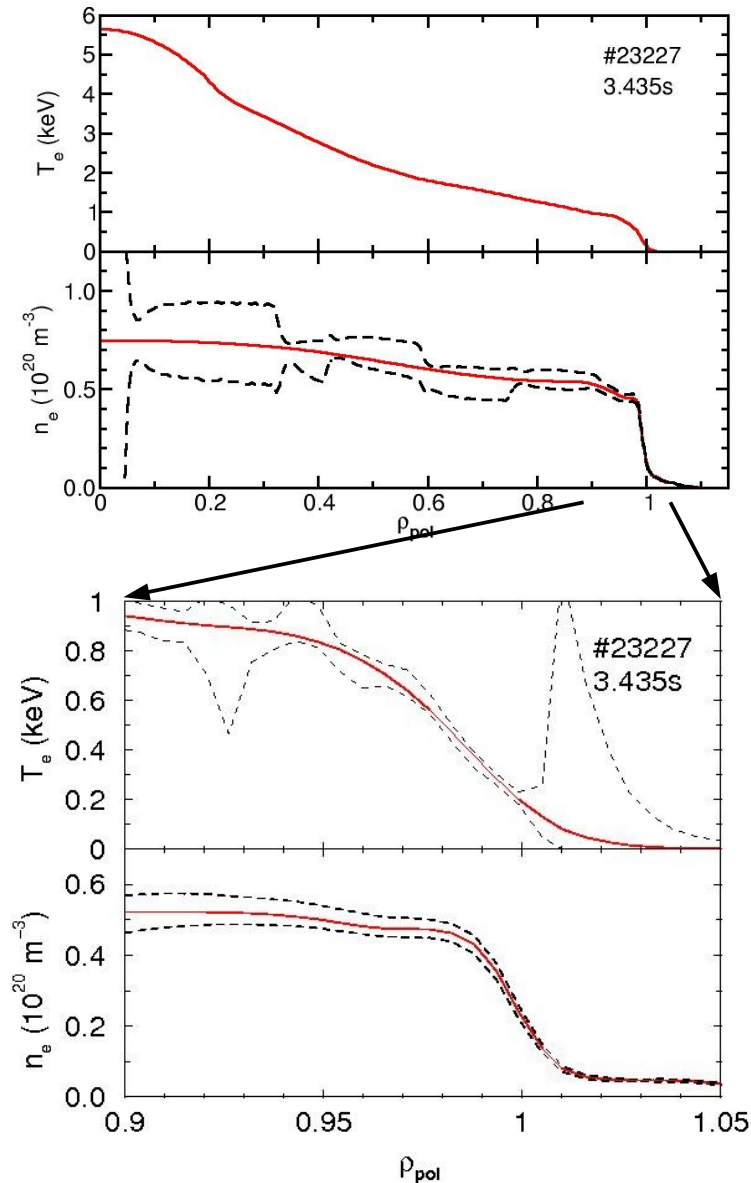
- optically thick plasma: blackbody radiation
- optically thin plasma: broadened emission region



$$d \propto T_e$$

$$d \propto f(T_e, n_e)$$



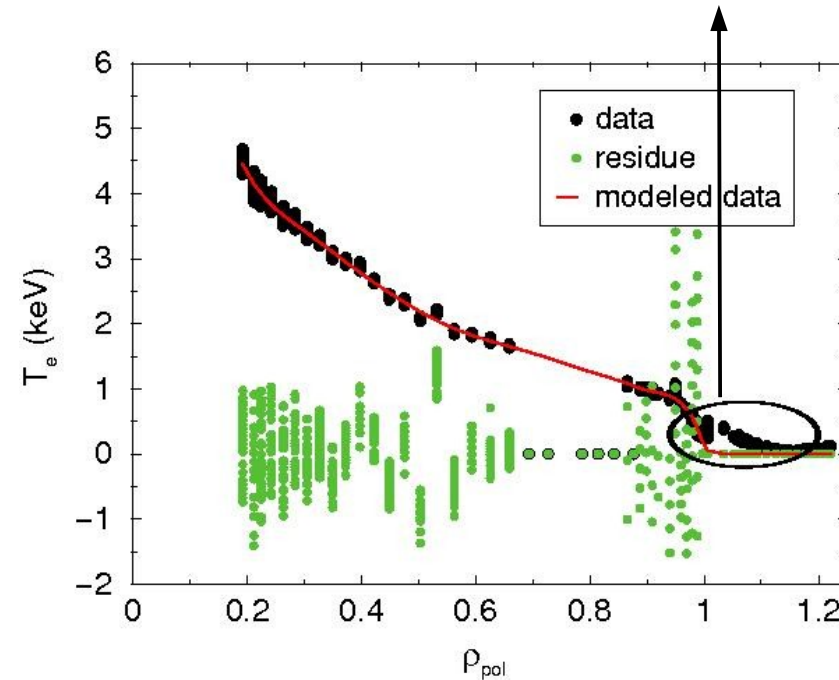


→ simultaneous:

- ✓ full density profiles
- ✓ (partly) temperature profiles
- ✓ pressure profile

→ $n_e > n_{e, \text{cut-off}}$ → masking of ECE channels

→ opt. depth $\sim n_e T_e$ → masking of ECE channels



IDA: LIB + DCN + ECE radiation transport

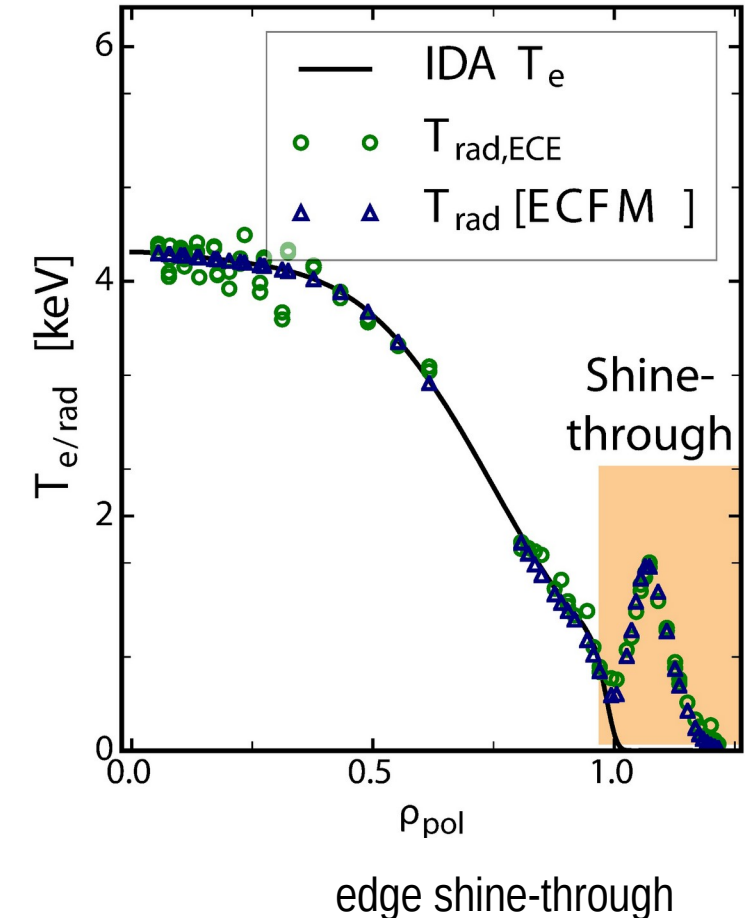
- Optically thick plasma: local emission and black-body radiation
- Optically thin plasma (edge and core)
 - EC emission depends on T_e and n_e
 - combination with data from density diagnostics is mandatory
 - calculate broadened EC emission and absorption profiles by solving the radiation transport equation

$$\frac{dI_\omega(s)}{ds} = j_\omega(s; n_e, T_e) - \alpha_\omega(s; n_e, T_e) I_\omega(s)$$

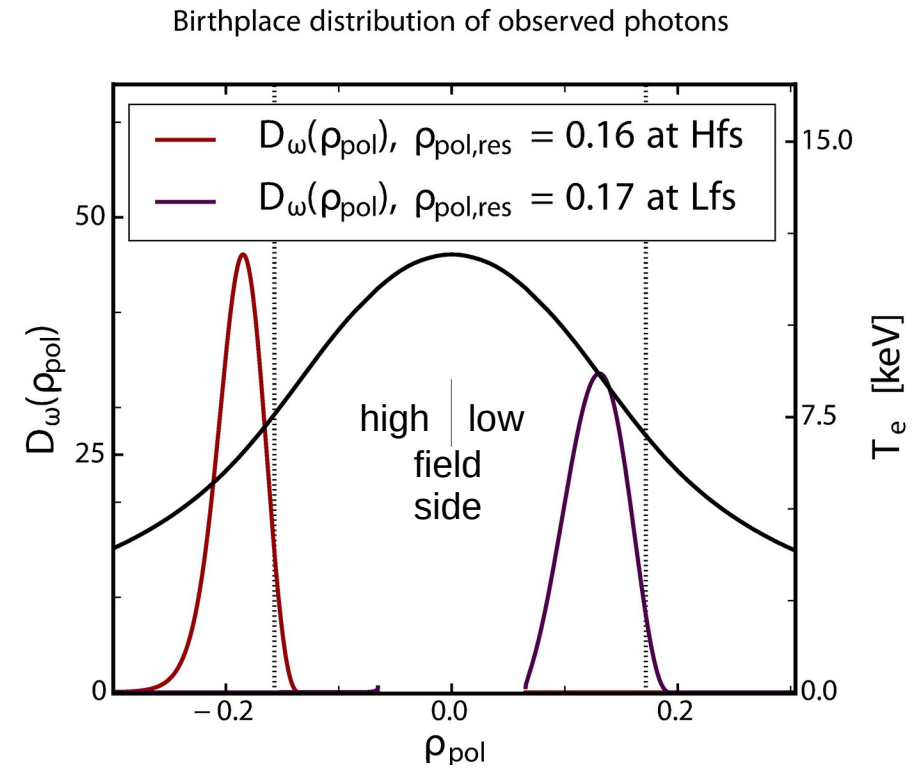
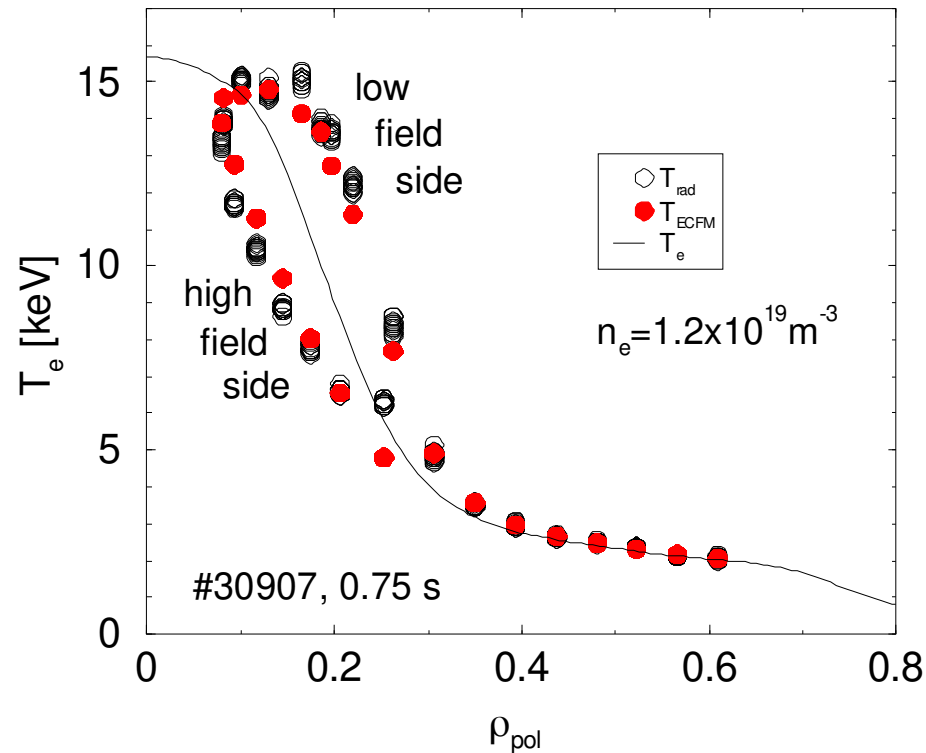
s LOS coordinate
 I_ω spectral intensity
 j_ω emissivity
 α_ω reabsorption

- electron cyclotron forward model (ECFM → ECRad) in the framework of Integrated Data Analysis
 (S.K. Rathgeber et al., PPCF 55 (2013) 025004; S.S. Denk et al., PPCF 60 (2018) 105010)

T_{Rad}, T_e for #30589 $t = 1.27$ s



IDA: ECE radiation transport → Core Shine-through



ECE core hfs-lfs loop if small optical depth in core (small n_e) and large core T_e -gradient:

- extended microwave emission region
- high-field side: $T_{\text{rad}} < T_e$ due to shine-through of smaller (outer) temperatures
- low-field side: $T_{\text{rad}} > T_e$ due to shine-through of larger (inner) temperatures

[“Non-thermal electron distributions measured with ECE”, S. Denk, Master thesis, 2014]

IDA: Uncertainties in Profiles (and Equilibria)



➤ How reliable are results from modeling codes (e.g. GENE, SOLPS, TGLF, ASTRA)?

uncertainty quantification (UQ), uncertainty propagation (UP), and validation (V)

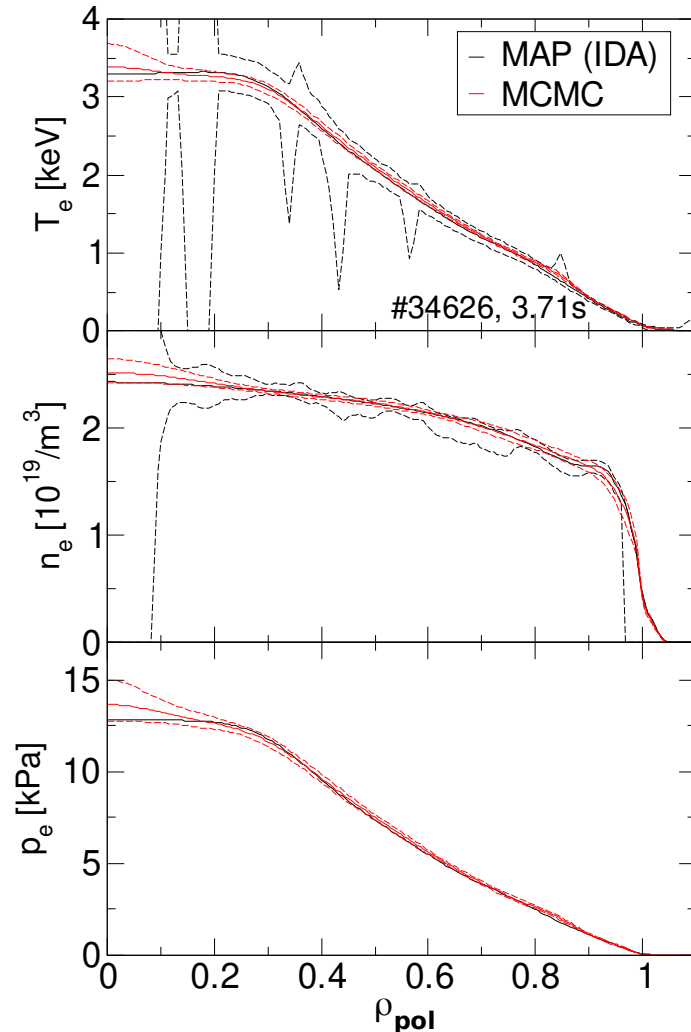
→ uncertain input quantities:

→ profiles ($T_e, n_e, T_i, n_i, Z_{\text{eff}}, v_{\text{rot}}, n_{\text{fast}}, \dots$) and uncertainties

→ profile gradients ($dT_e/dR, dT_e/d\rho, d\ln(T_e)/d\rho, \dots$) and uncertainties

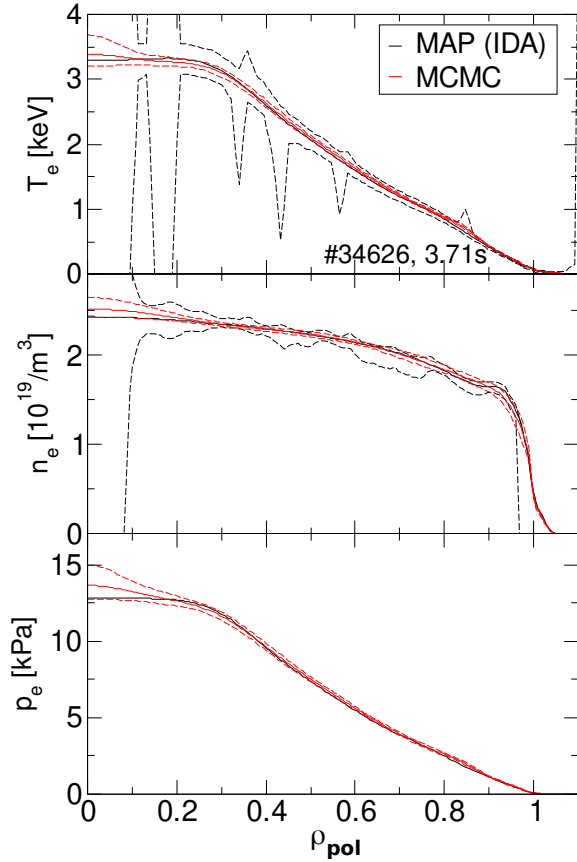
→ equilibrium and uncertainties

IDA: Uncertainties in Profiles: MAP, MCMC

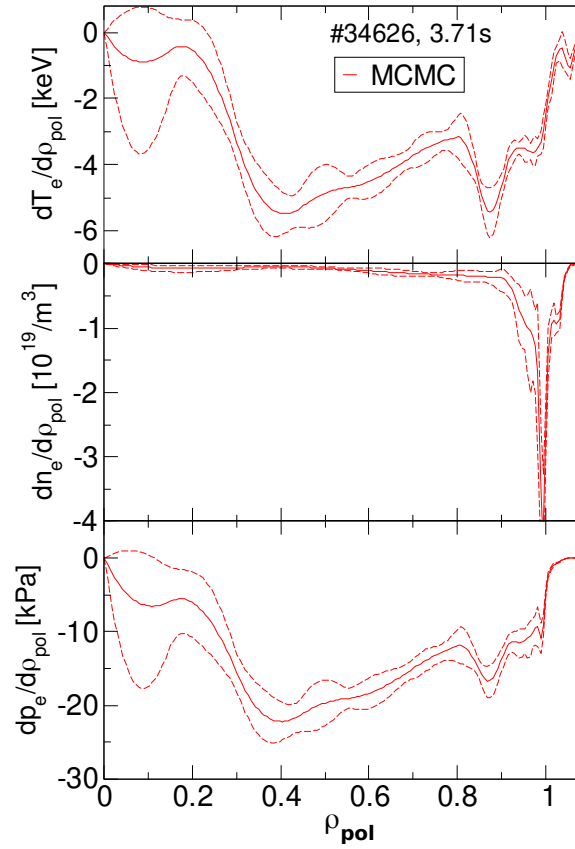


- **Maximum a Posteriori (MAP)**
 - (T_e, n_e) estimate
 - error bar
 1. Covariance at probability maximum → local quantity
 2. from local profile changes and effect on χ^2 (Fischer, PPCF 2008) without profile correlations
- **Markov chain Monte Carlo (MCMC) sampling of posterior pdf**
 - mean → (T_e, n_e) estimate
 - variance → (small) error bar (incl. correlations)
 - profile samples
 - error propagation in modeling codes

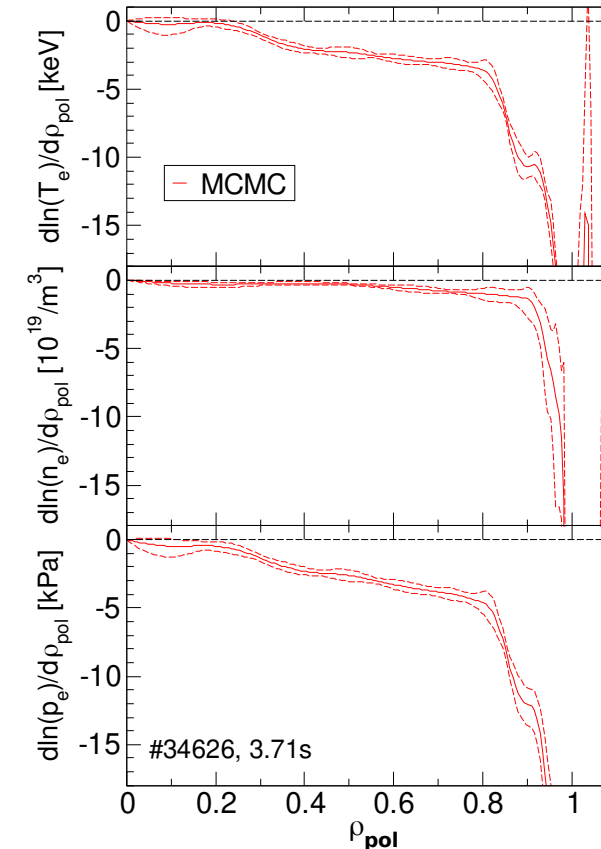
IDA: Logarithmic Gradients and Uncertainties



profiles

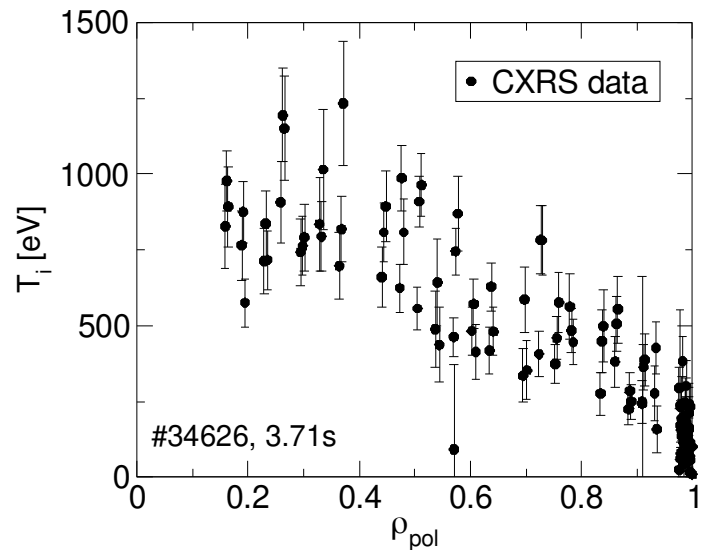


profile gradients



logarithmic profile gradients

→ GENE: UQ and UP (F. Jenko, C. Michoski, Univ. Texas Austin)



R.M. McDermott, RSI 2017

- interpolation and smoothing of noisy data
- uncertainties of profiles and profile gradients
- extrapolation to axis

- Gaussian process (GP): random variable has (multidimensional) normal distribution

- GP regression (GPR):

→ no assumption about profile shape!

→ different positions are correlated depending on distance

→ likelihood

$$\text{Cov}(f_k, f_l) = \eta^2 \exp\left(-\frac{(x_k - x_l)^2}{2\xi^2}\right)$$

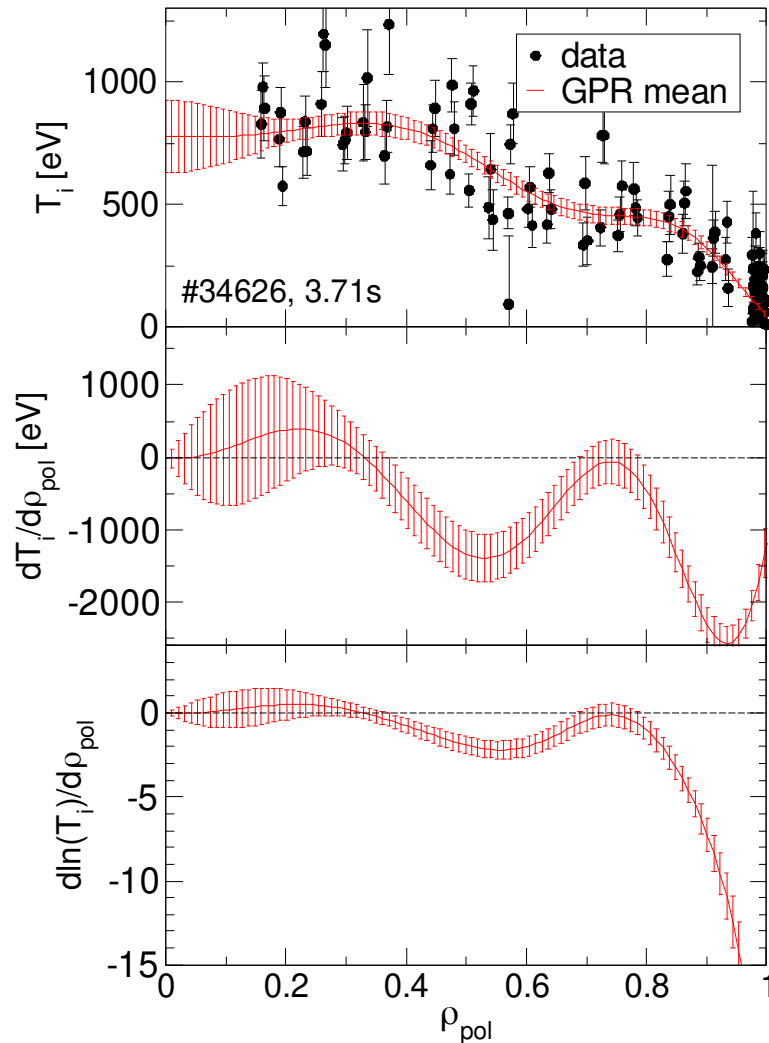
$$p(\vec{d} | f(x), \vec{\sigma}) = N(\vec{d} | f(x), \vec{\sigma})$$

- Result: (Gaussian) probability distribution of possible interpolating functions

→ Mean/variance of pdf: Analytic solution for profile, gradients and their uncertainties

→ Samples of pdf: candidate profiles to study UP in modeling codes

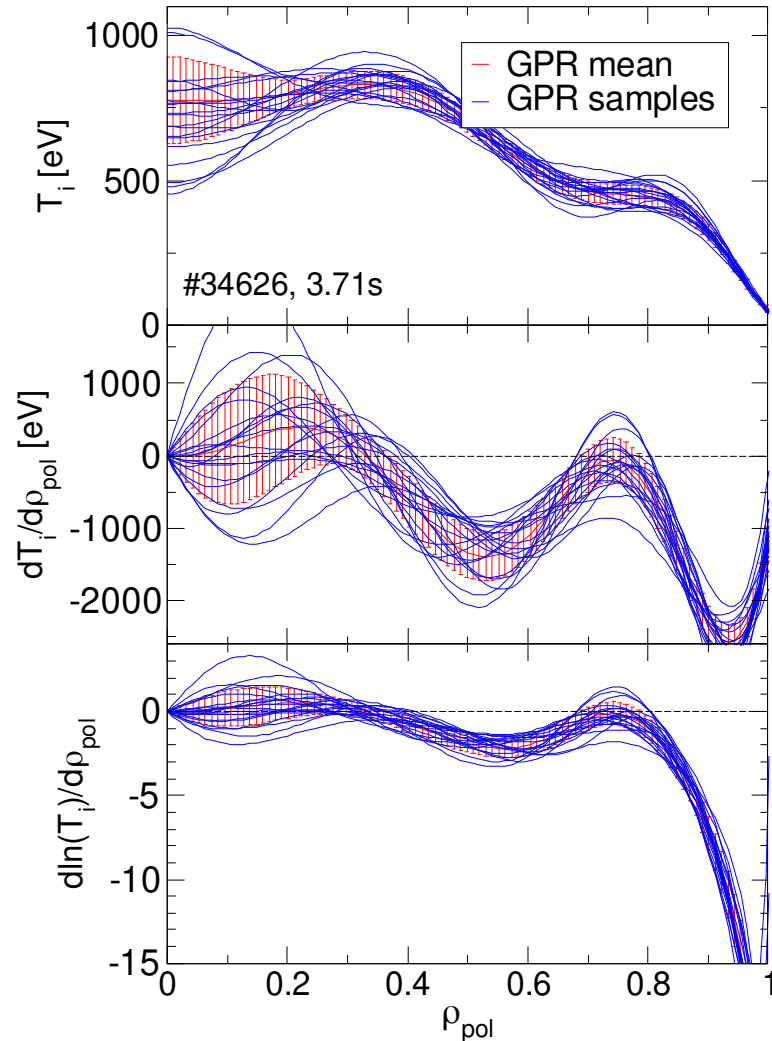
GPR: Profile, Gradient and Uncertainty



- estimation and uncertainty (1 std) of
 - T_i profile
 - T_i profile gradient
 - T_i profile logarithmic gradient
- result depends on parameters:
 - correlation length ξ (might depend on position ρ_{pol} : *non-stationary*)
 - $\xi \downarrow \rightarrow$ uncertainty $\uparrow$
 - kernel weight η
- constraint $dT_i/d\rho_{pol}=0$ at magn. axis

→ GENE: UQ and UP (F. Jenko, C. Michoski)

IDA: Samples of Profiles and Gradients



- samples of
 - T_i profile,
 - T_i profile gradient and
 - T_i profile logarithmic gradient

useful for uncertainty propagation (UP)
in modeling codes

- mean and uncertainty of
profiles, gradients, logarithmic gradients
and covariance matrices for sampling
 - fast: $\sim 6s$ / 1000 time points
 - also for v_{tor}

The IDE equilibrium solver reconstructs the current distribution by solving the

1. **Grad-Shafranov equation:** Ideal magnetohydrodynamic equilibrium for poloidal flux function Ψ for axisymmetric geometry

$$\left(R \frac{\partial}{\partial R} \frac{1}{R} \frac{\partial}{\partial R} + \frac{\partial^2}{\partial z^2} \right) \Psi = -(2\pi)^2 \mu_0 (R^2 P' + \mu_0 F F')$$

subject to all available measured data (magnetics, pressure profile, polarimetry, (i)MSE, (SOL-)tile currents, loop voltages, iso-flux constraints, ...)
and (non-physical) smoothness constraints to regularize the ill-conditioned solver

coupled with the

2. **Current diffusion equation:** describes the diffusion of the poloidal flux Ψ on the background of the toroidal flux $\Phi(\rho)$ due to resistivity

$$\sigma_{\parallel} \frac{\partial \Psi}{\partial t} = \frac{R_0 J^2}{\mu_0 \rho} \frac{\partial}{\partial \rho} \left(\frac{G_2}{J} \frac{\partial \Psi}{\partial \rho} \right) - \frac{V'}{2\pi\rho} (j_{bs} + j_{ec} + j_{nb})$$

Goal: replace non-physical smoothness constraints by a temporal correlation defined by the current diffusion

IDA: Magnetic Equilibrium: Sawtooth Crash

Evolution of current distribution between sawtooth crashes:

Most important ingredients:

1. GSE:

+ pressure profiles: $p_e + p_i + p_{\text{fast}}$
(+ polarimetry)
(+ MSE and iMSE)

2. CDE (neoclassical current diffusion):

+ kinetic profiles ($\rightarrow$ conductivity $\rightarrow T_e$):

$T_e, n_e \rightarrow$ IDA (central ECE!)

$T_i \rightarrow$ GPR

$n_i \sim f(n_e - n_{\text{fast}}); Z_{\text{eff}}$ (bremsstrahlung)

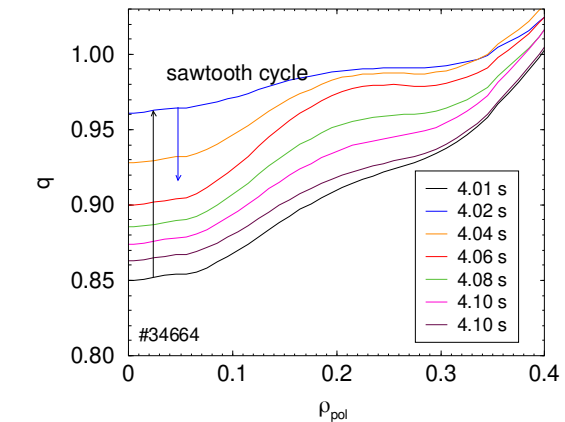
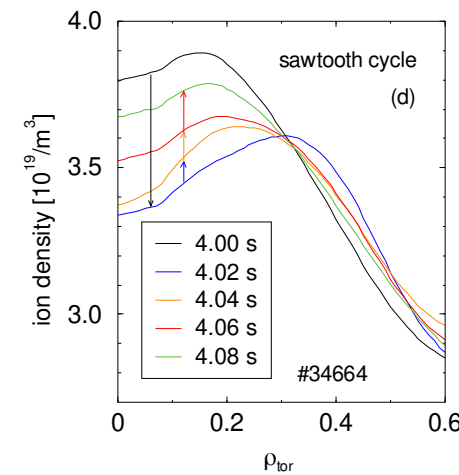
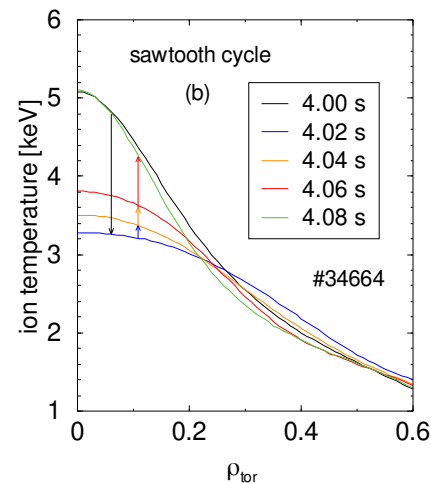
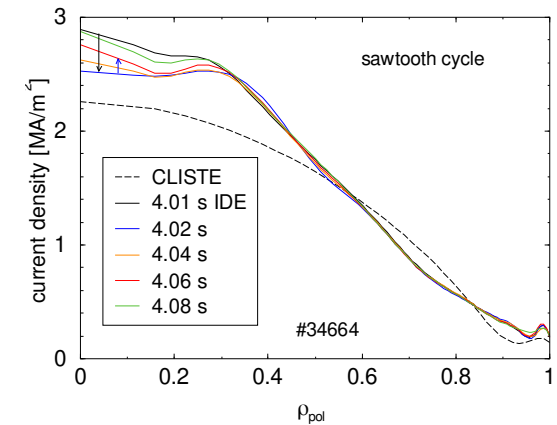
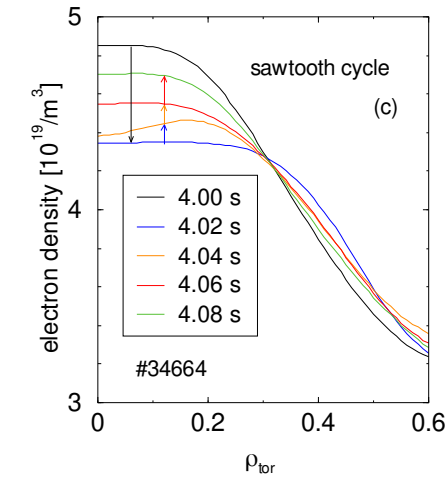
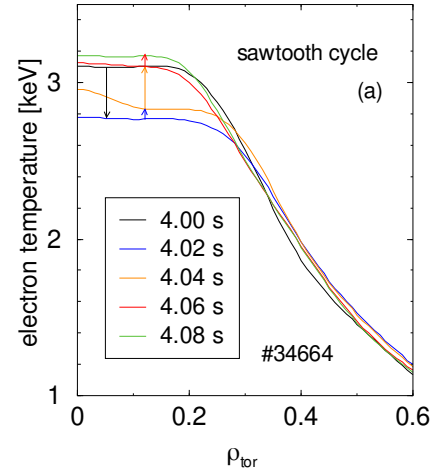
+ j_{ECCD} from TORBEAM

+ j_{NBCD} from RABBIT (Weiland NF 2018)

3. Sawtooth times (soft X-ray)

and current relaxation model

(Kadomtsev or FCM (Fischer NF 2019))



Further Applications of IDA



W7-AS: n_e, T_e : TS, interferometry, soft X-ray

ASDEX UG: n_e, T_e : TS, interferometry, ECE, ...

Z_{eff} : bremsstrahlung spectra

T_i, v_{rot} : CXRS

equilibrium: Grad-Shafranov, current diffusion, many diagnostics

W7-X: non-Maxwellian electron energy distribution function: visible emission spectrum

$n_e, T_{e/i}$, impurity densities, flows: TS, X-ray imaging

n_e, T_e : TS, interferometry, helium beam

Z_{eff} : bremsstrahlung spectra

MST RFP: T_e : TS, soft X-ray

Z_{eff} : soft X-ray, CXRS

TJ-II: n_e, T_e : TS, interferometry, reflectometry, Helium beam

JET: n_e : LIB

n_e, T_e : LIDAR, interferometry

fast-ion distributions : velocity-space tomography of fast-ion D-alpha spectroscopy, collective TS, gamma-ray and neutron emission spectrometry, and neutral particle analyzers.

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D. Dodt et al., J. Phys. D: Appl. Phys., 41:205207, 2008.

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S. Kwak et al., arXiv:2103.07582, 2021

S. Kwak et al., RSI, 92:043505 (2021)

L. M. Reusch et al., RSI, 85:11D844, 2014.

M.E. Galante et al., NF, 55:123016, 2015.

B. Ph. van Milligen, et al., RSI 82, 073503 (2011)

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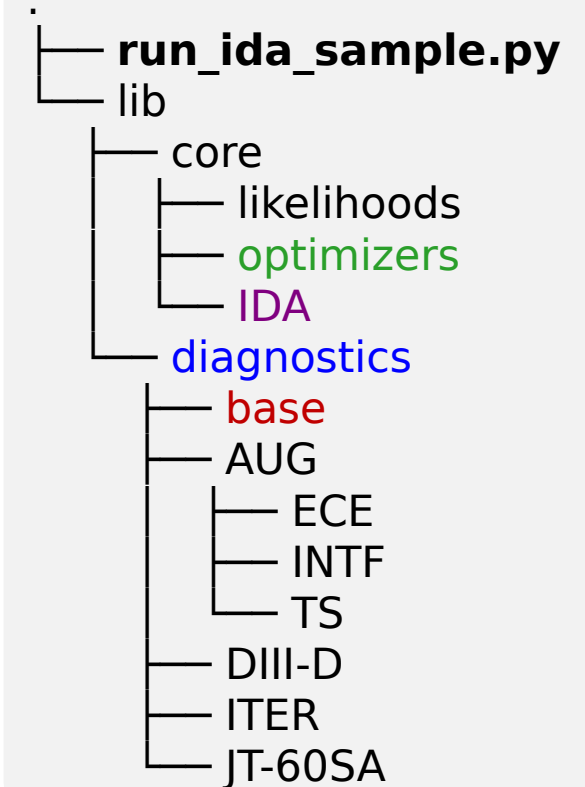
M. Salewski et al., FST 74:23–36, 2018.

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arXiv:2411.09270 [physics.plasm-ph]*

IDA Basic Implementation for ITER, JT-60SA, ...

Basic implementation in python being completely modular

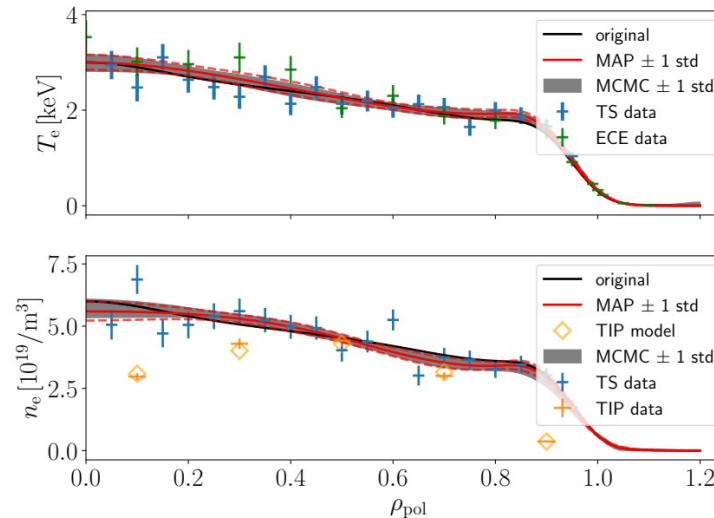
- to be compatible with any fusion device (ITER, DIII-D, JT-60SA, ...)
- **diagnostics**: Thomson scattering, ECE and interferometry, ...
- **likelihoods** (data uncertainty): Gaussian, Cauchy (outlier robust), ...
- **multi-fidelity forward models** / synthetic diagnostics
 - ECE: $T_{\text{rad}} = T_e$ vs radiation transport modeling $T_{\text{rad}}(T_e, n_e)$
 - real-time vs offline analysis
- flexible **parameterisation** of, e.g., profiles: splines, GPR, ...
- **priors**: smoothness, positivity, physical modeling, ...
- **results and their uncertainties**:
 - MAP solution (probability maximum and width)
 - MCMC sampling methods (explore full probability space)



IDA: ITER workflow

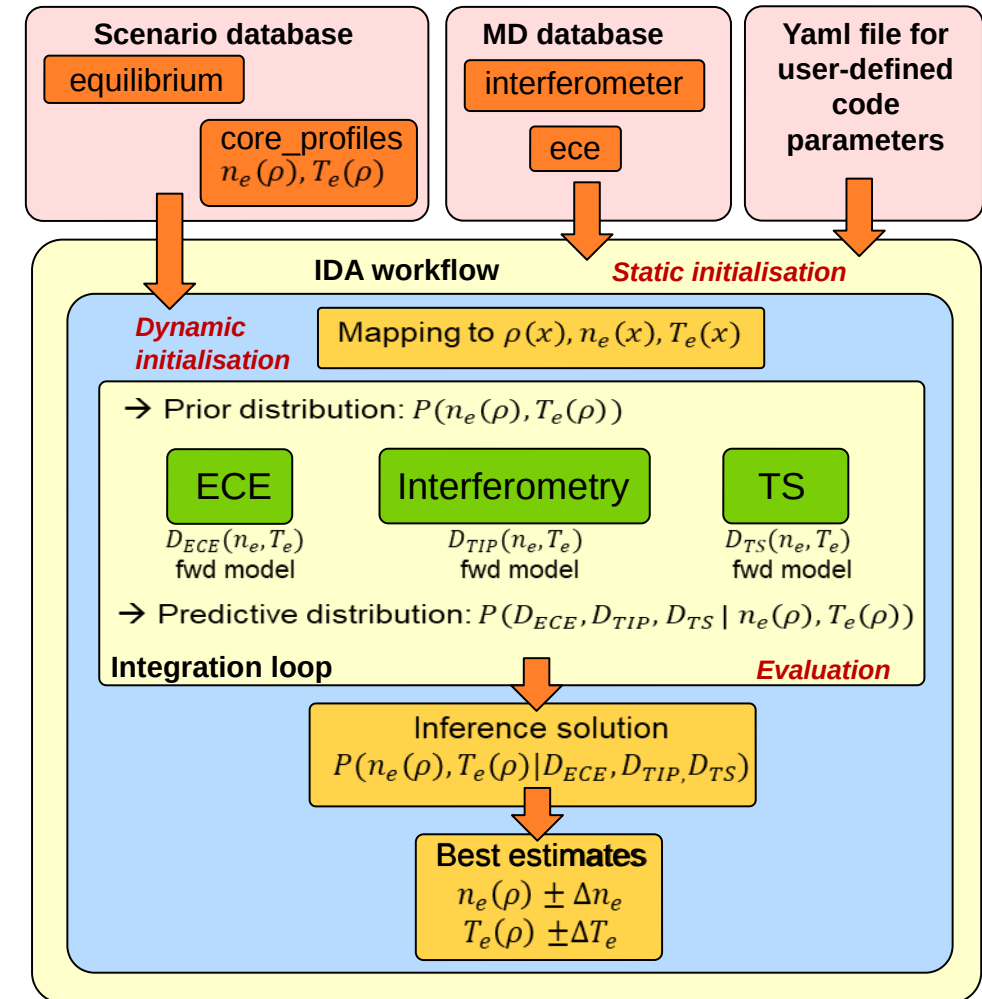
- artificial diagnostics: Thomson scattering, ECE
 - synthetic data set with 10% noise
- 1st ITER diagnostic: Toroidal Interferometer Polarimeter (TIP)
 - synthetic data set with 5% noise
 - IMAS synthetic diag.

- MAP ± 1 std
- MCMC (50 $\pm$ 34)% percentile



IMAS Interface Data Structures (IDS):

- read: TIP geometry (interferometer_md), equilibrium
- write: results ...



M. Schneider

IDA in the Bayesian framework for Nuclear Fusion



Bring together different **diagnostics/diagnosticians/theoreticians** with **redundant/complementary/modeling** data

- **Probabilistic modeling of individual diagnostics** (forward models, likelihoods for all kind of uncertainties)
- **Probabilistic combination of different diagnostics** (multiply pdfs, unified error analysis, error propagation)
- **Probabilistic combination with prior / modeling information**
- **Redundant** data:
 - more reliable results by larger (meta-) data set → reduction of estimation uncertainties
 - detect and resolve data inconsistencies (reliable/consistent diagnostics) using standardized error/uncertainty treatment
- **Complementary** data:
 - resolve parametric entanglement
 - resolve complex error propagation (non-Gaussian)
 - synergistic effects (exploiting full probabilistic correlation)
 - automatic *in-situ* and *in-vivo* calibration (transient effects, degradation, ...)
 - advanced data analysis technique → improvements in modeling (ECE) and diagnostics hardware (LIB)
- **Goal**: Coherent combination of measurements from different diagnostics
 - **replace** combination of **results** from individual diagnostics
 - **with** combination of **measured data** → one-step analysis of pooled data
 - in a **probabilistic** framework

*The actual science of logic is conversant at present only with things either certain, impossible, or entirely doubtful, none of which (fortunately) we have to reason on. Therefore **the true logic** for this world **is the calculus of Probabilities**, which takes account of the magnitude of the probability which is, or ought to be, in a **reasonable man's mind**.*

James Clerk Maxwell (1850)