Some unsolved challenges in RF current drive

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Unsolved problems include current drive in magnetic geometries in which the toroidal magnetic field cannot be assumed to be dominant, current start-up with hyper-resistivity, current drive with oscillating parameters, and synergistic effects between current drive and alpha channeling. These problems are not necessarily straightforward to solve, but there is the potential of significant consequence.

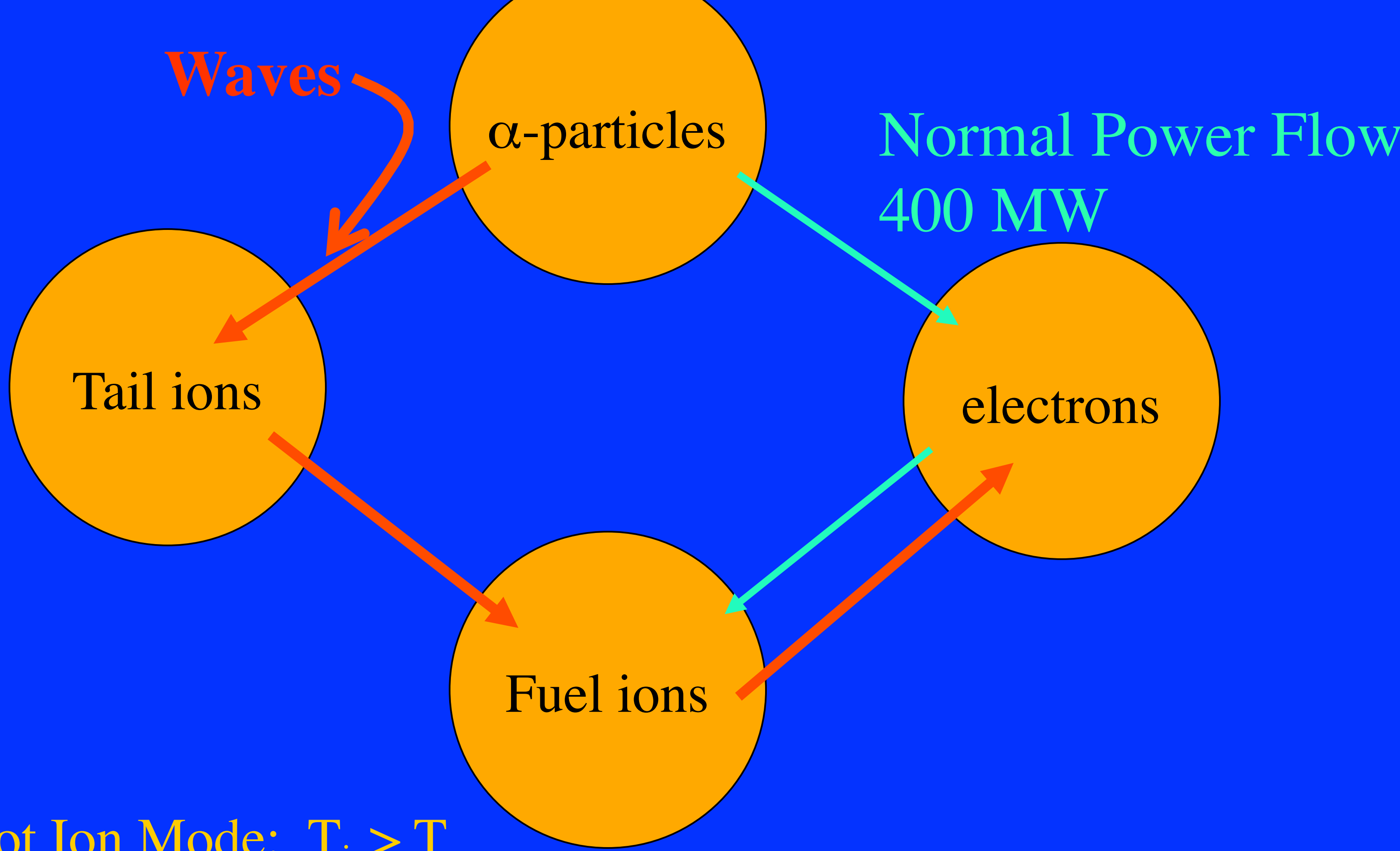
Some Unsolved Challenges in RF Heating and Current Drive

1. Alpha Channeling – How to accomplish?
2. Current Drive effects associated with ions
3. Neoclassical pinch effects associated with trapped electrons
4. High-Efficiency Cyclic Operation
5. Combine with Alpha Channeling: Engineering and Physics
6. Can Hyper-resistivity be induced?
7. Free Energy *Complexity Theory*

Power Flow in a Fusion Reactor

Advantages of “α-Channeling”

$D + T \rightarrow He^4 + n$



Get Hot Ion Mode: $T_i > T_e$

75% of α power to ions $\Rightarrow P_f \rightarrow 2\,P_f$

Fisch and Rax, 1992

Reactor designs around Aries I operating point

	no channeling		channeling	
	cd	P	75%	75%
T_i(keV)	20	15	20	15
T_e(keV)	20	15	12	12
$n(10^{14}\ cm^{-3})$	1.2	1.8	1.8	2.1
τ_i(s)	2.0	2.0	2.0	1.0
τ_e(s)	1.0	0.7	0.3	0.5
$P_f\ (W\ cm^{-3})$	4.7	6.1	10.9	9.7

Fisch and Herrmann, 1994

Advantages of Alpha Channeling

1. Because of the increased reactivity at a given confined pressure (and the free current drive), the hot ion mode gives about 30% cheaper COE, compared to aggressively designed reactors.
2. The impurities can be removed and the plasma can be fueled easily.
3. However, it may be more desirable yet, if electron heat transport is not tamed. Ion transport might eventually be tamed, but maybe not electron transport, in which case having ions hotter than electrons reduces the heat loss substantially.
4. The present data base of the top tokamak confinement and heating results supports hot-ion mode operation only.

Hot-ion Mode RF-Driven Tokamak

Are we prepared?
Perhaps the eventual reactor will be in the hot ion mode.

Does equal temperature mode really extrapolate?

1. Top performance results to date (JET, TFTR) achieved in hot ion mode.
2. Perhaps ion heat transport will be well-controlled but not electron heat transport

Upside to hot ion mode -- better extrapolation and 30% on COE

1. RF energy channeled from alpha particles
2. Fusion reactivity can be doubled in hot ion mode.
3. RF current drive fueled by alpha channeling.
4. Ash removal. Fueling.
5. Expedited by possible resonant "ringing" of tokamak.
6. Electron heat can be poorly confined.
7. Less free energy to drive instabilities.

If the reactor will be in the hot ion mode, then expect

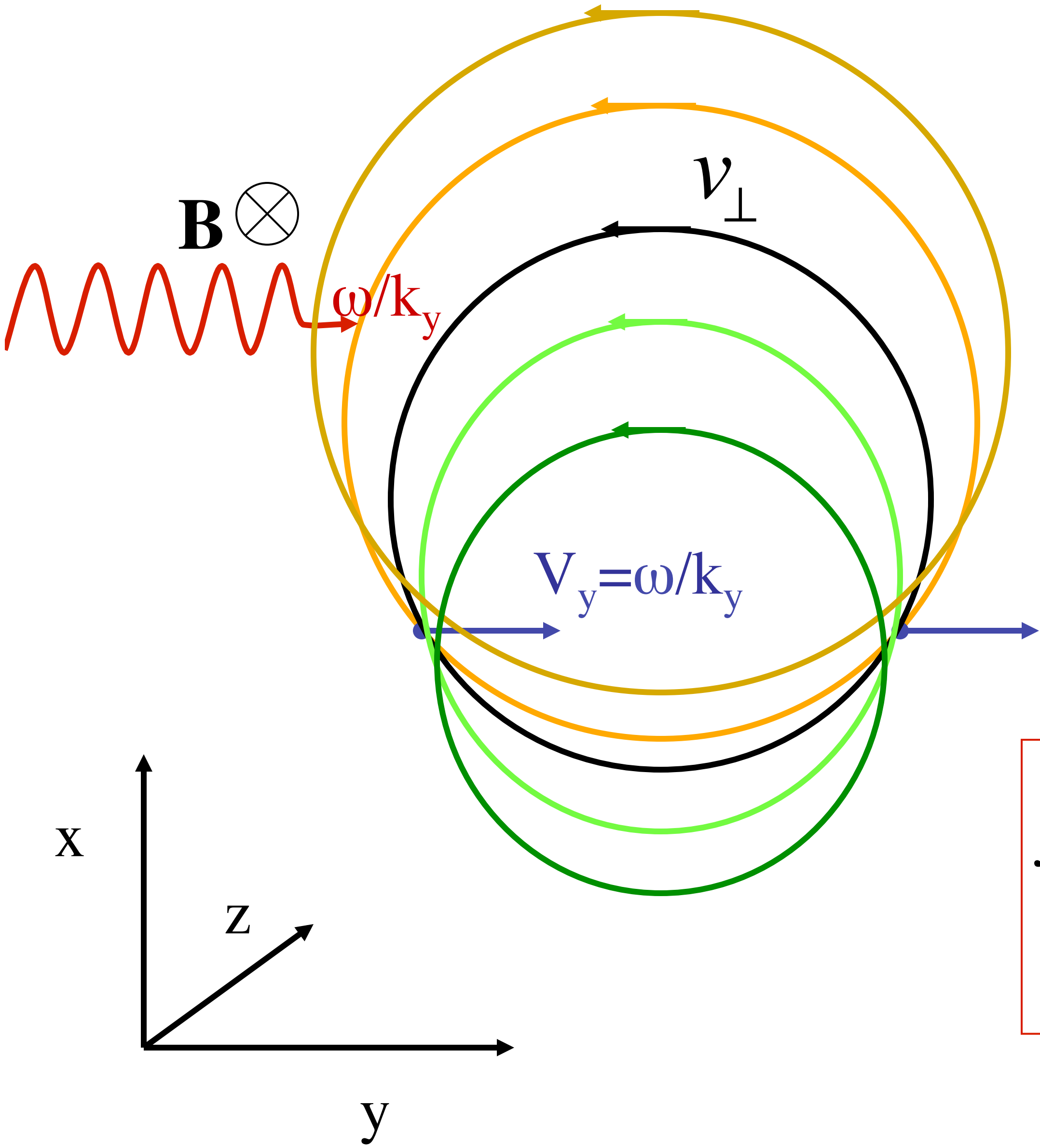
Highly rf-driven reactor, possibly with 400 MW or more RF, where RF is first-order physics.

RF-Driven Tokamak

A more essential role for rf physics, technology, and modeling

1. Steady state achieved by rf current drive for much of the current.
2. Control of transport: plasma fueling and ash removal
3. Rf energy channeled from alpha particles.
4. Resonant “ringing” of tokamak!
5. Highly rf-driven reactor, possibly with 400 or more MW rf, where rf is first-order physics.
6. Non-issues: alpha-driven instabilities, poor electron heat confinement or poor alpha particle radial flux.

Diffusion Paths



$$v_y \rightarrow v_y + \Delta v_y$$

$$x_{gc} \rightarrow x_{gc} + \Delta v_y / \Omega$$

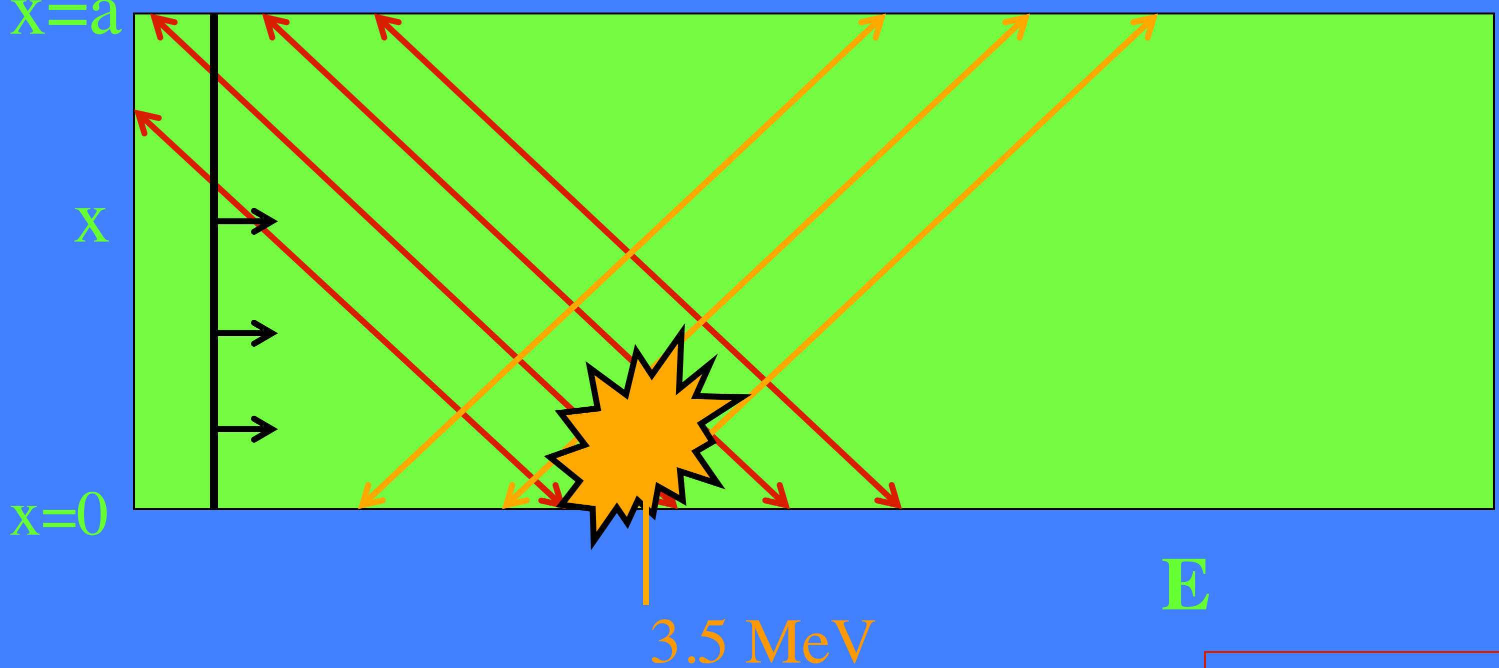
$$\Omega \equiv eB/m$$

$$\Delta E = m v_y \Delta v_y$$

$$\boxed{x_{gc} \rightarrow x_{gc} + \frac{\Delta E}{m\Omega \, \omega / k_y}}$$

Fisch and Rax, 1992

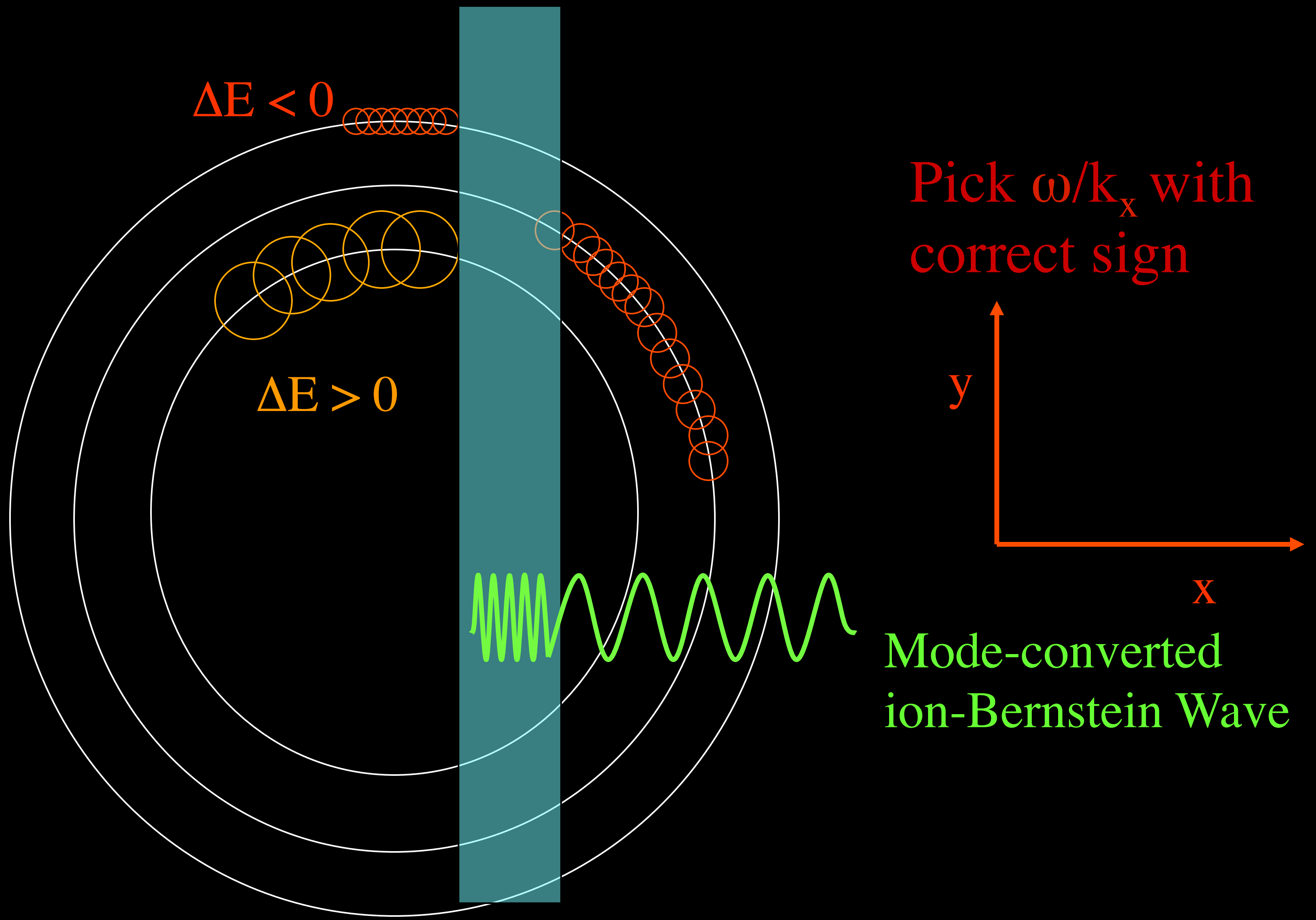
Diffusion Paths



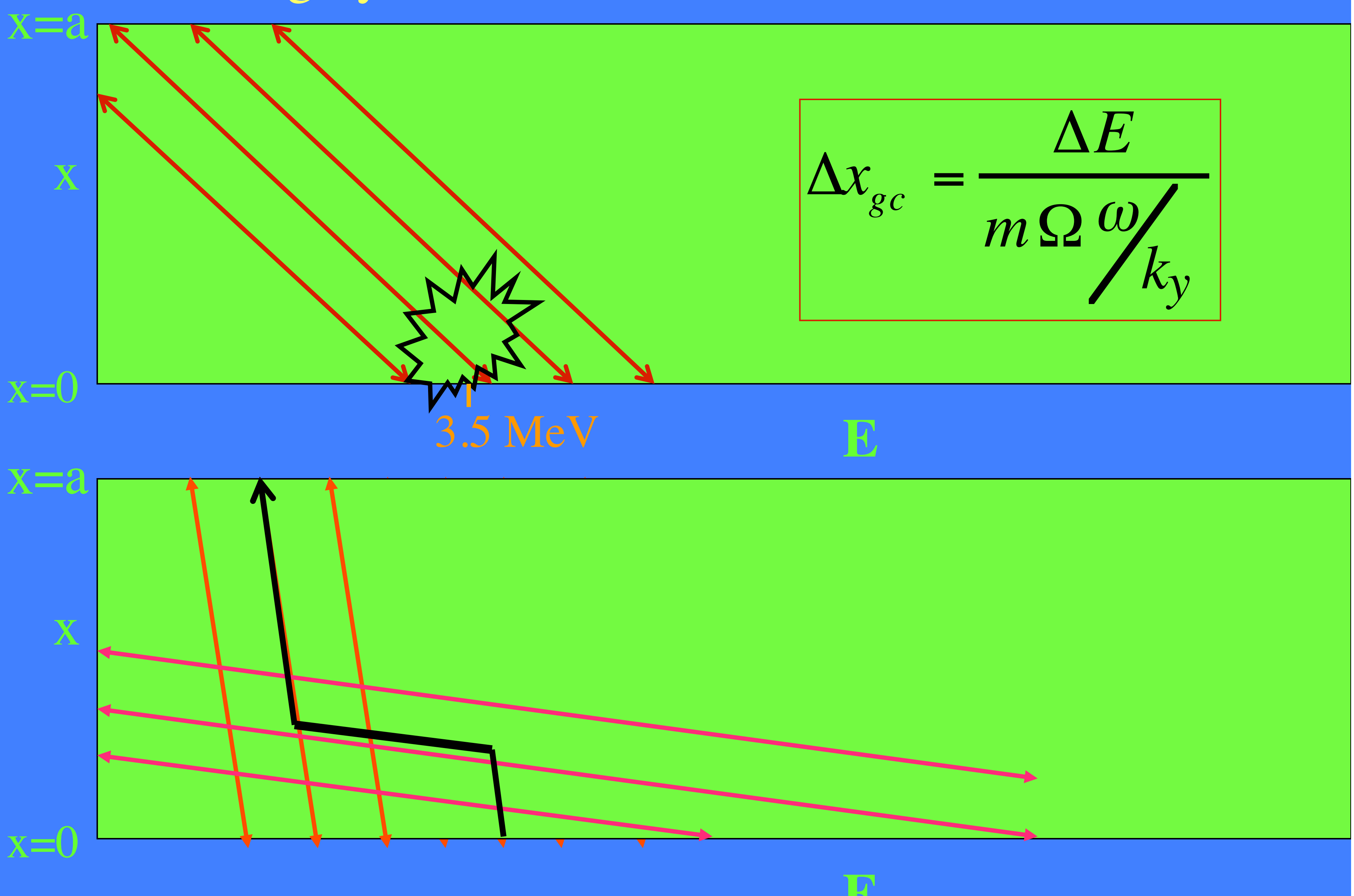
$$\Delta x_{gc} = \frac{\Delta E}{m\Omega \, \omega / k_y}$$

$$\omega - \vec{k} \cdot \vec{v} = 0$$

Tapping Free Energy in α-Particles

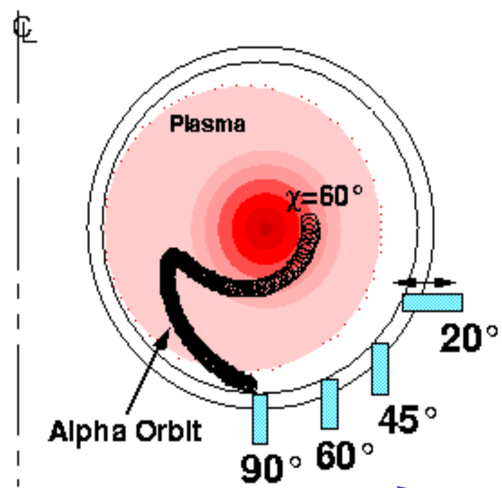


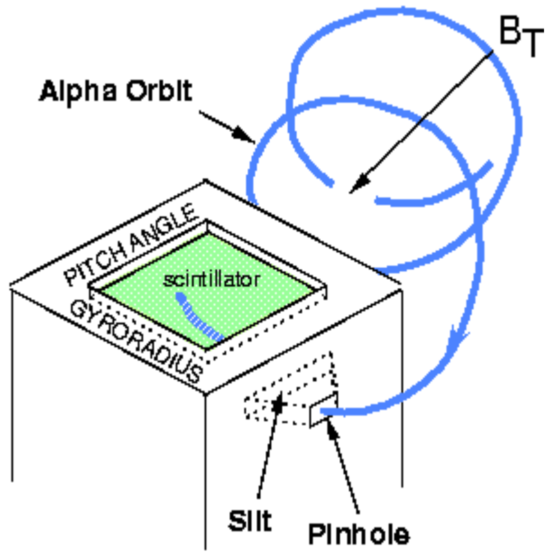
Diffusion Paths with 2 Waves: Highly Constrained Stochastic Motion



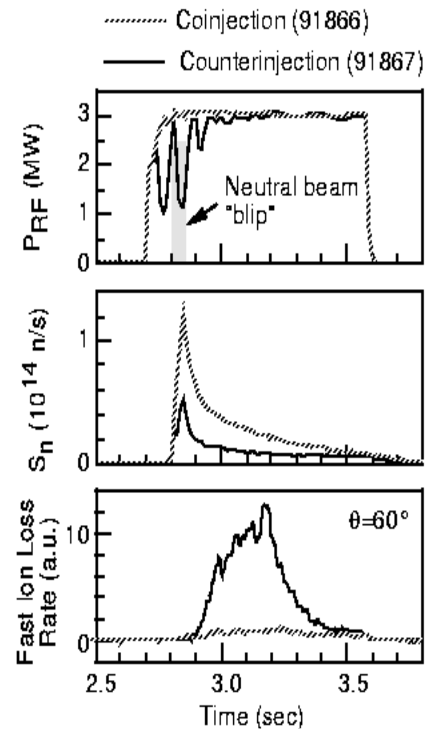
$$\Delta x_{gc} = \frac{\Delta E}{m\Omega \, \omega / k_y}$$

TFTR D-Beam MCIBW Experiments





Zweben's lost alpha detectors



Darrow, 1996

4D information: energy, poloidal angle, pitch angle, time!

Experimental Results

Fisch et al. (1996), Herrmann and Fisch (1997)

1. Key characteristics of mode-converted IBW verified on TFTR

2. Detailed verification of diffusion (phased for heating) in E-μ-P_ϕ space

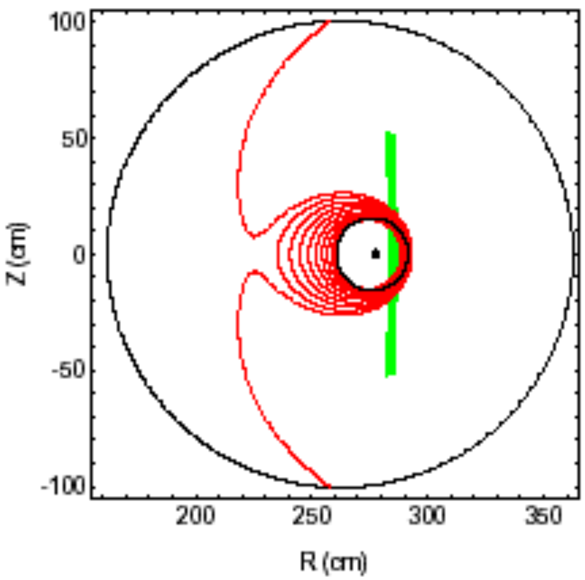
Absolute value of diffusion coefficient appears to be factor of 50 higher than simple ray-tracing theory implies!

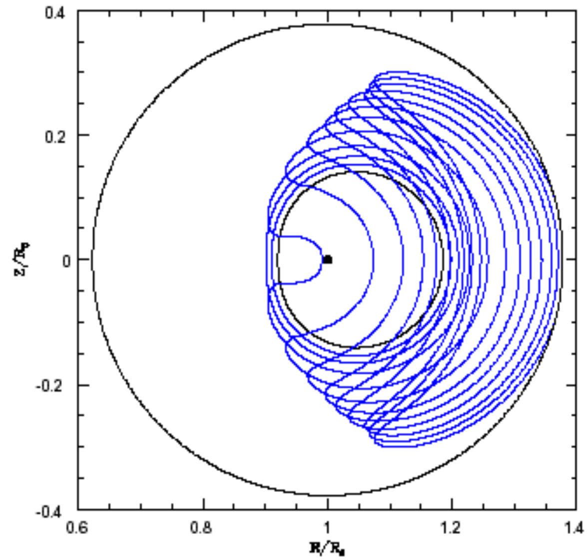
Possibility of Exciting Internal Mode

Clark and Fisch, 2000

⇨ Perhaps α-channeling effect can be achieved even at low power

Wave-Particle Interaction Physics





$$\omega - k_{\|} v_{\|} = \Omega_\alpha \qquad \omega - n_\varphi \omega_\varphi - m\omega_\theta = 0$$

$$\frac{dP_\varphi}{dE} = \frac{n_\varphi}{\omega} \qquad \frac{dP_\varphi}{dE} = \frac{n_\varphi}{\omega}$$

$$\frac{d\mu}{dE} = \frac{e}{m\omega} \qquad \frac{d\mu}{dE} = 0$$

Destiny of trapped electrons under LHCD

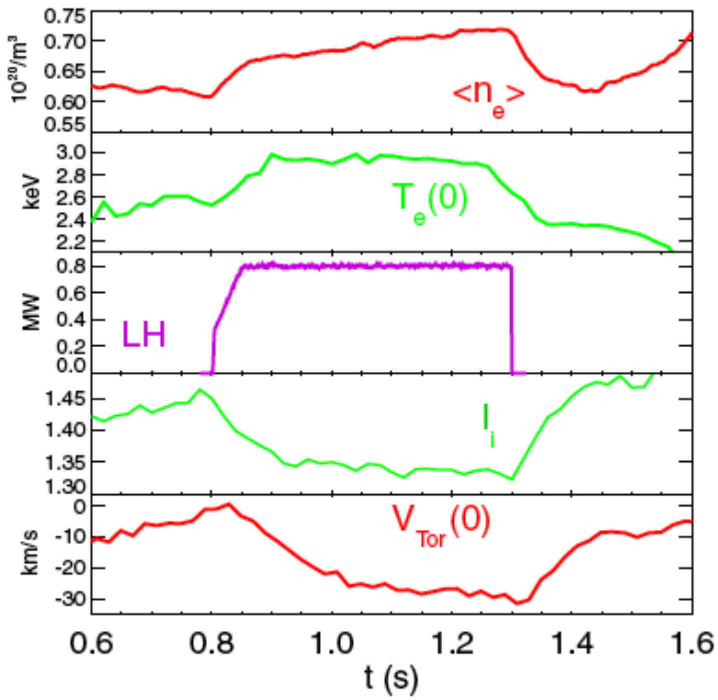
1. RF Pinch effect: Canonical angular momentum of trapped electrons is conserved, so electrons must change vector potential upon absorbing wave momentum (Fisch and Karney, 1981).
2. But electron pinch is not a steady state option.
3. Do rotation measurements (Rice, 2008) inform on destiny of trapped electrons through radial electric field?
4. More generally, an open challenge is LHCD in ST. Use 5D.
5. Suggestion: Use adjoint formalism to calculate Green's function response for linear absorption.

MIT LHCD Experiment (2008)

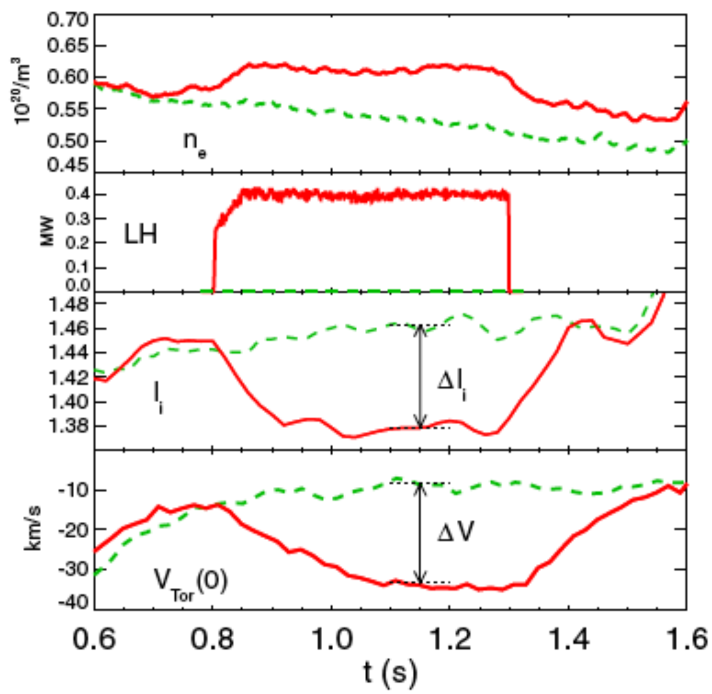
Counter-Current Core Toroidal Rotation with LHCD

Counter-current increment in V_ϕ
similar drop in internal inductance
(broadening of current profile)
peaking of central electron density
peaking of core electron and ion temperature

Time scale for V_ϕ evolution much longer than τ_E or τ_ϕ (~35 ms) but similar to the current relaxation time, τ_{CR} ~ 250 ms.



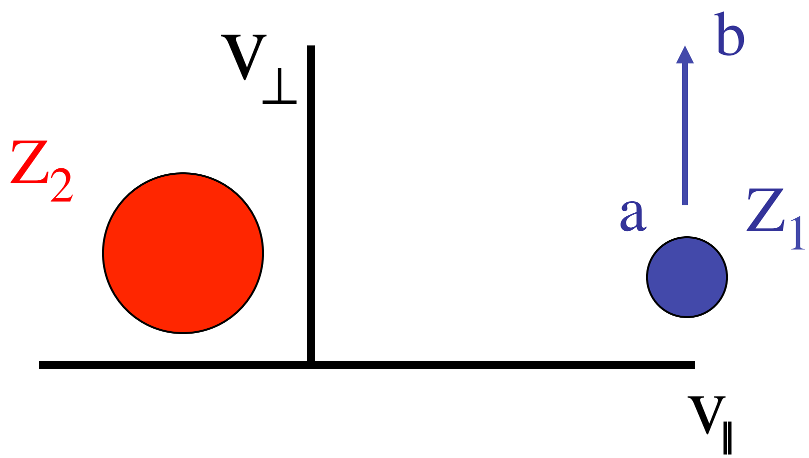
Comparison of two discharges, with and without LHCD



Ince-Cushman et al (2008), Rice et al (NF, 2009)

Minority ion Cyclotron Current Drive Effect (MiCCD)

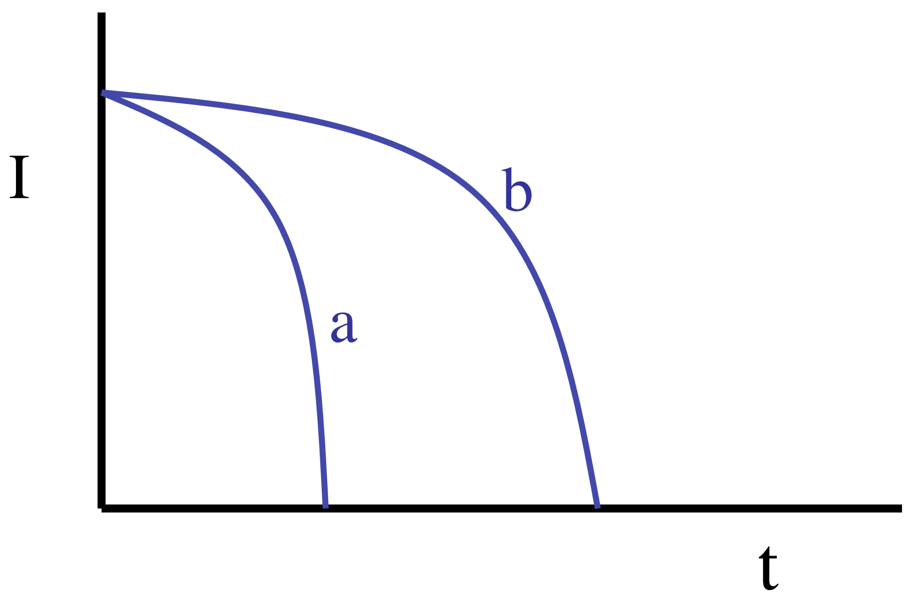
Use waves to get counter-streaming ions



Zero ion current

Non-Zero electron current

if $Z_2 \neq Z_1$



$$\omega - \vec{k} \cdot \vec{v} = n\Omega$$

$$\Omega \equiv eB/m$$

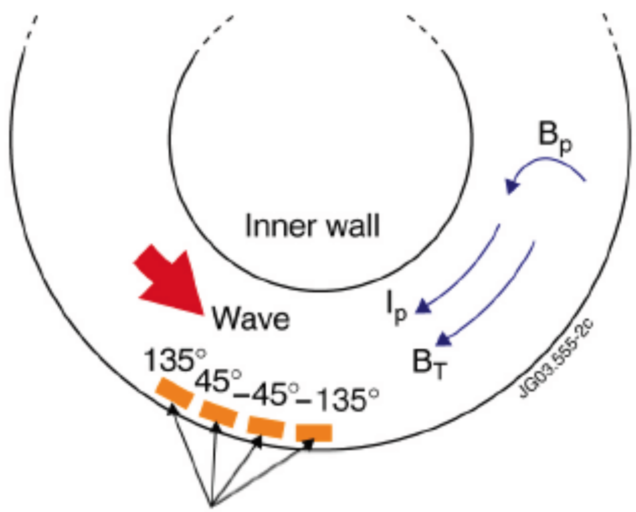
Fisch, 1981

Minority ion Cyclotron Current Drive

EFDA EUROPEAN FUSION DEVELOPMENT AGREEMENT JET

Sawtooth control by ICRF waves
Ion Cyclotron Current Drive

- How to affect sawtooth :
  - ✓ High central fast particle pressure: stabilisation
  - ✓ Local modification of current profile at R_{INV} : destabilisation or stabilisation
- Destabilisation by ICRF waves
  - ✓ Toroidally directed ICRF waves : co or counter current propagation
  - ✓ Finite $k_{//} \Rightarrow$ asymmetric coupling to ions and electrons in $v_{//}$-space $\Rightarrow$ dipolar ICCD *(Fish, Nucl. Fusion, 1981)*
- Effect will depends on:
  - ✓ Toroidal direction of the wave
  - ✓ Location of R_{res} versus R_{inv}



Note: MiCCD effect is complicated if
1. absorption straddles resonance
2. Ion trapping

Thus, other current drive effects may dominate

Hellsten et al., PRL, 1995
Carlson et al., PP, 1998

LHCD Power Required

$$\frac{P_{\mathrm{rf}}}{P_f} \simeq \frac{15}{J/P_d} \frac{1}{(n_{14} T_{10} a_1 R_1)^{1/2} (3T_{10} - 2)}$$

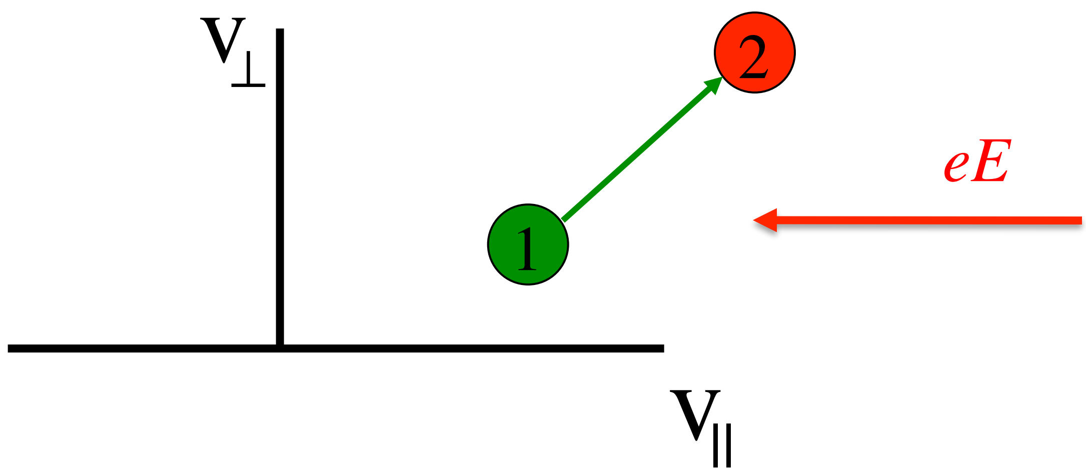
Design 1 (small and cold)	Design 2 (large and hot)
$T_{10} = 1$	$T_{10} = 2$
$n_{14} = 1$	$n_{14} = \frac{1}{3}$
$a_1 = 3$	$a_1 = 5$
$R_1 = 8$	$R_1 = 13$
$P_f \simeq 1.8$ GW	$P_f \simeq 3.3$ GW
$H \simeq 1.5$ MW/m^2	$H \simeq 1$ MW/m^2
$\frac{P_{\mathrm{rf}}}{P_f} \simeq \boxed{\frac{15}{J/P_d}}$ 20%	$\frac{P_{\mathrm{rf}}}{P_f} \simeq \boxed{\frac{15}{J/P_d}}$ 3%

Tokamak Recharge or "Oscillating Current Drive"

Suppose that current is generated under one set of conditions, but then allowed to relax under a different set of conditions. What is the current-drive efficiency?

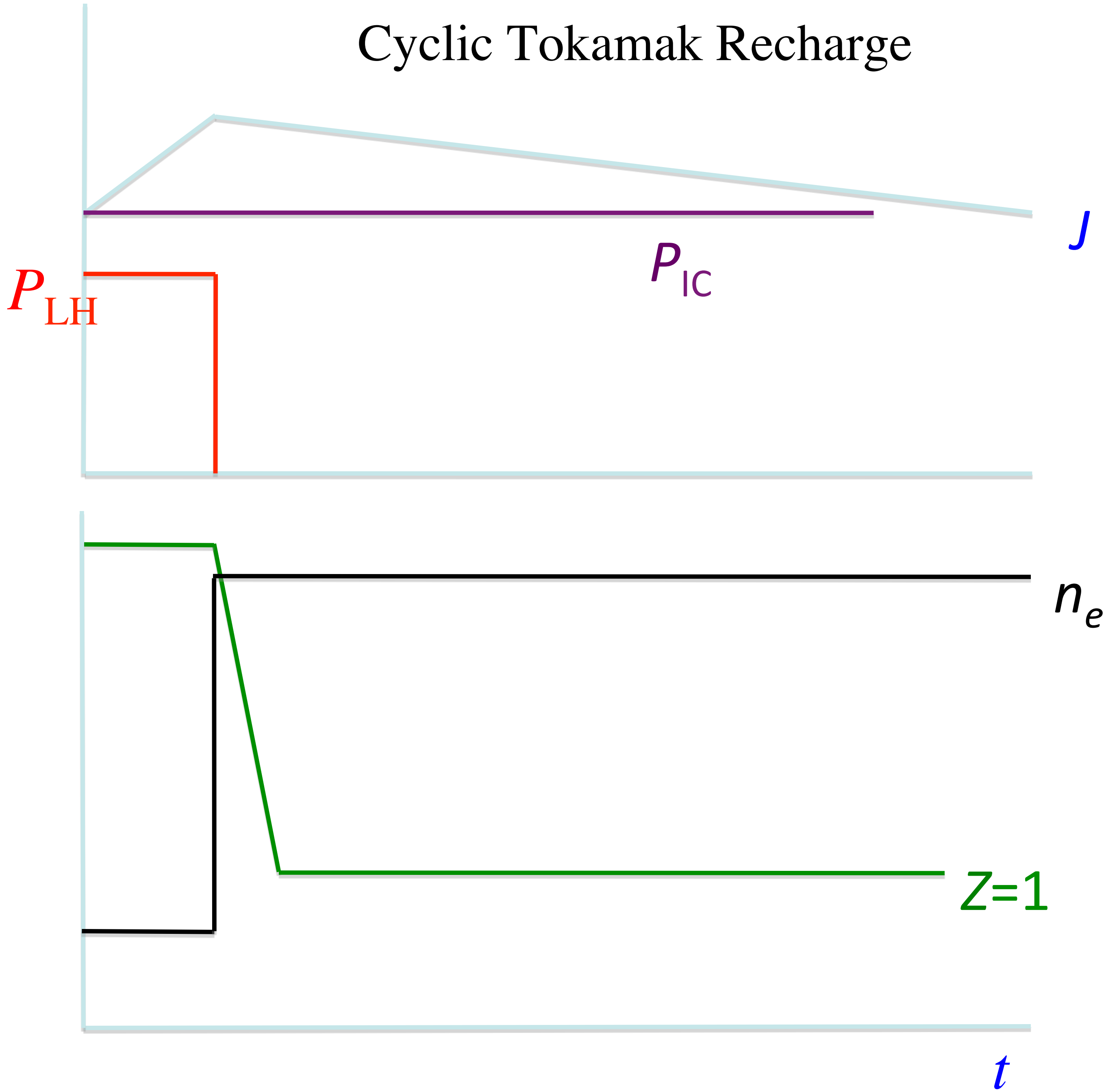
Take advantage of separation of time scales: $\tau_c \ll J/(dJ/dt) \ll L/R$

$$\frac{dJ}{dt} + \frac{J}{\tau_g} = \frac{J_{rf}}{\tau_g} \qquad \frac{dJ}{dt} + \frac{J}{\tau_r} = 0 \qquad E = \eta\left(J - J_{rf}\right)$$



Fisch, 1987

Cyclic Tokamak Recharge



compare: Peysson, Bae, Calabro (this conference)

Cyclic Operation

$$J_{\max} - J_{\min} \approx J_{\max} \frac{T_r}{\tau_r} \approx J_{rf} \frac{T_g}{\tau_g}$$ periodic

$$J_{\max} \approx J_{\min} \approx J_0$$ Minimum variation in J

$$\frac{T_r}{T_g} \approx \frac{J_{rf}\tau_r}{J_0\tau_g} >> 1$$ maximize relaxation stage

$$W_c \equiv P_d T_g \approx \frac{J_{rf}}{\left\langle J/P_d \right\rangle_g} T_g$$ Energy dissipated

$$\left\langle J/P_d \right\rangle_{avr} \cong W_c/\tau_r \cong \left\langle J/P_d \right\rangle_g \frac{\tau_r}{\tau_g}$$ Average efficiency

$$\frac{\tau_r}{\tau_g}\ ,\ \frac{T_r}{T_g}\ ,\frac{J_{rf}}{J_0} >> 1$$

Optimizing Current Drive Efficiency

$$\boxed{\left\langle J/P_d \right\rangle_{LH} = \frac{-e\,\mathrm{v}_{\|}^{\,2}}{\nu_0(5+Z_i)}}$$

$$\left\langle J/P_d \right\rangle_{avr} \cong \left[J/P_d\right]_{Z=1} \frac{\tau_r}{\tau_g(Z=1)} \frac{6Z_g}{5+Z_g}$$

Note: factors are multiplicative, so get factor of say several in high-Z and further factor of several in low-*n*. Could also get factor of several by cycling T_e. The higher peak power required is balanced by the higher current drive efficiency to the extent that low-*n* is utilized.

Synergies with Alpha Channeling

Open question: What are the engineering implications?

Current Generation Stage

Low density to optimize current drive efficiency
Low T_e and high Z_{eff} to minimize σ

But these are just the conditions for hot ion mode!
Also, these are also conditions for proven LHCD

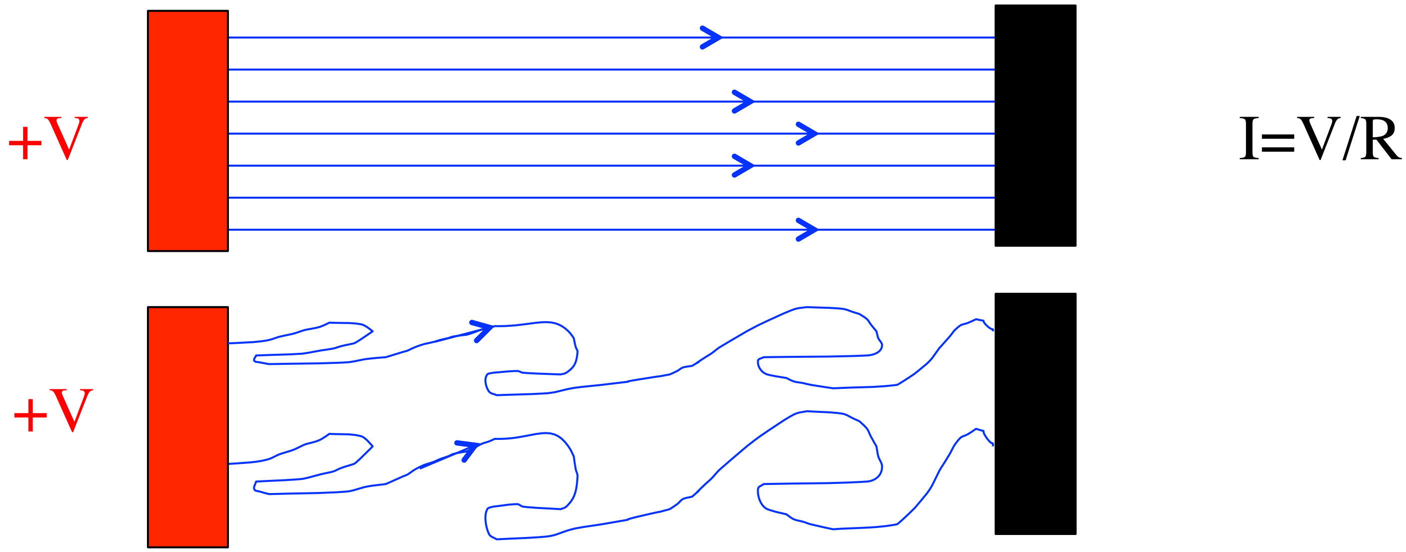
Two Possible optimizations

1. Optimize for current drive only, say $T_e = T_i$=5 keV, $n_e = 10^{13}$. In this case LHCD can be used efficiently.
2. Optimize for both current drive and fusion production via hot ion mode, say T_e = 15 keV, T_i=30 keV, $n_e = 10^{13}$. In this case, LHCD can be used to drive the current, and ion heating can separately provide the hot ion mode, or, better yet, a different wave mechanism can produce both the current and the hot ion mode via alpha channeling.

Fisch, JPP (2010)

RF-induced Hyper-resistivity?

(probably absurd, but it would be very useful)



If current carriers follow field lines, can one add to field line length to increase resistivity?

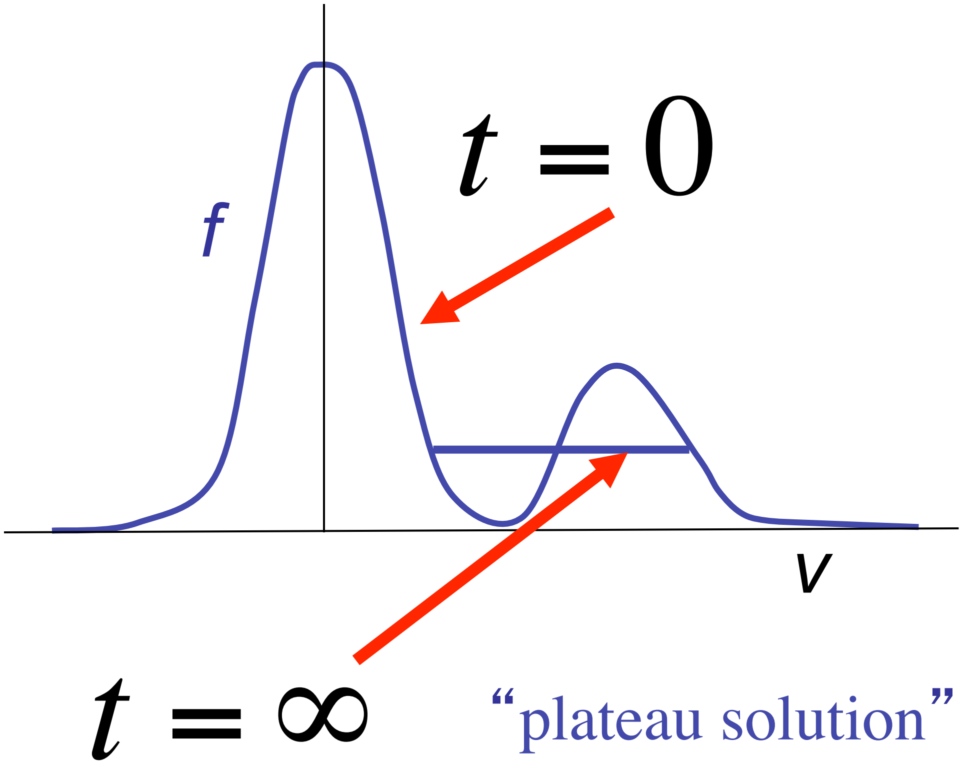
If so, is there a fundamental maximum increase?

Shouldn’t there be a limit to the length after which fine scales are averaged over?

If so, wouldn’t the resistivity increase be larger for Ohmic currents?

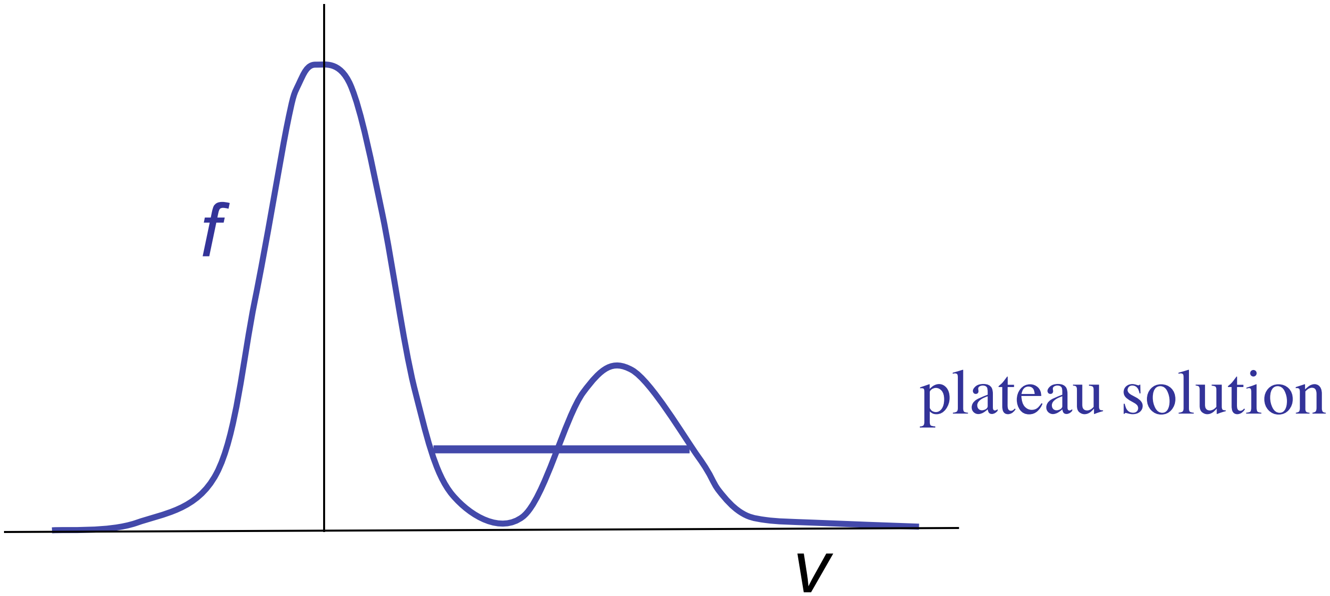
What would be the cost of increasing the resistivity?

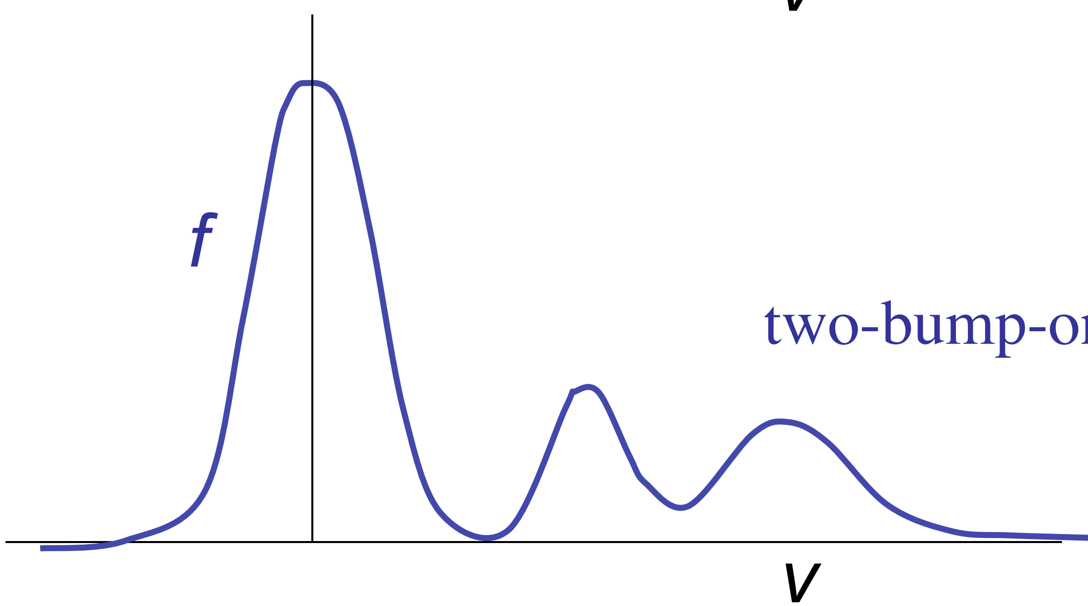
“Bump-on-tail” Instability



Free energy is due to equalizing population inversion
Diffusion over resonant wave region: Not entropy conserving
Recall Lecture by Professor J. M. Rax

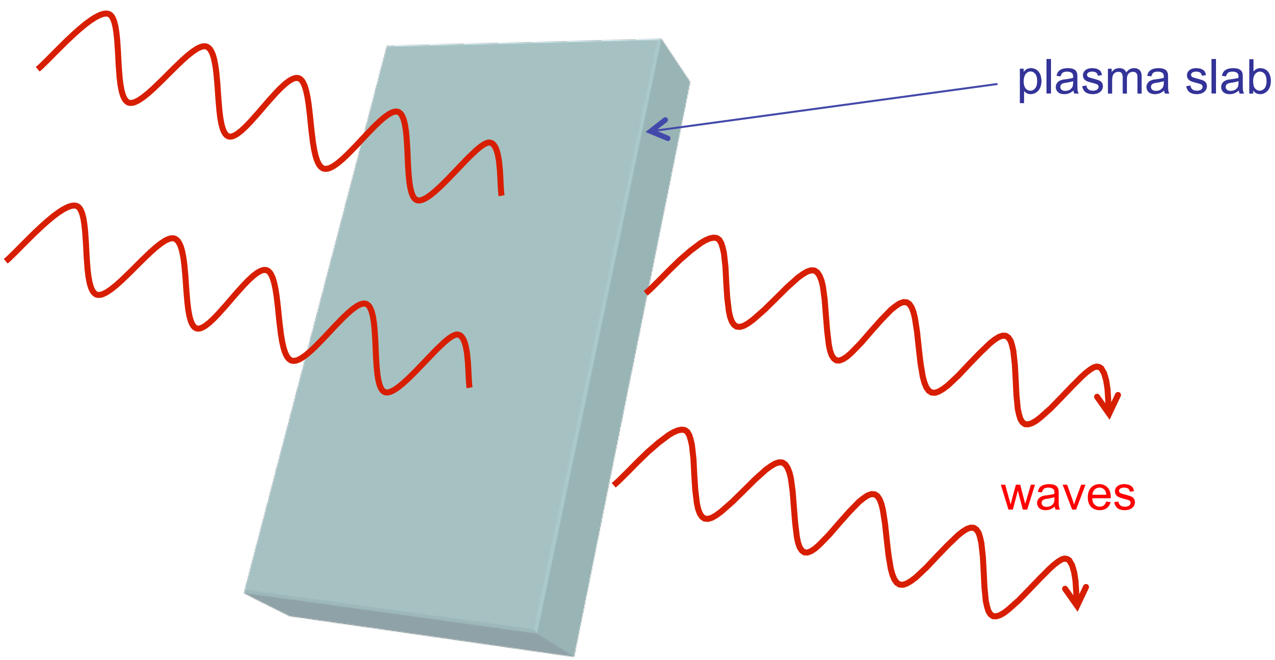
"Bump-on-tail" Instability





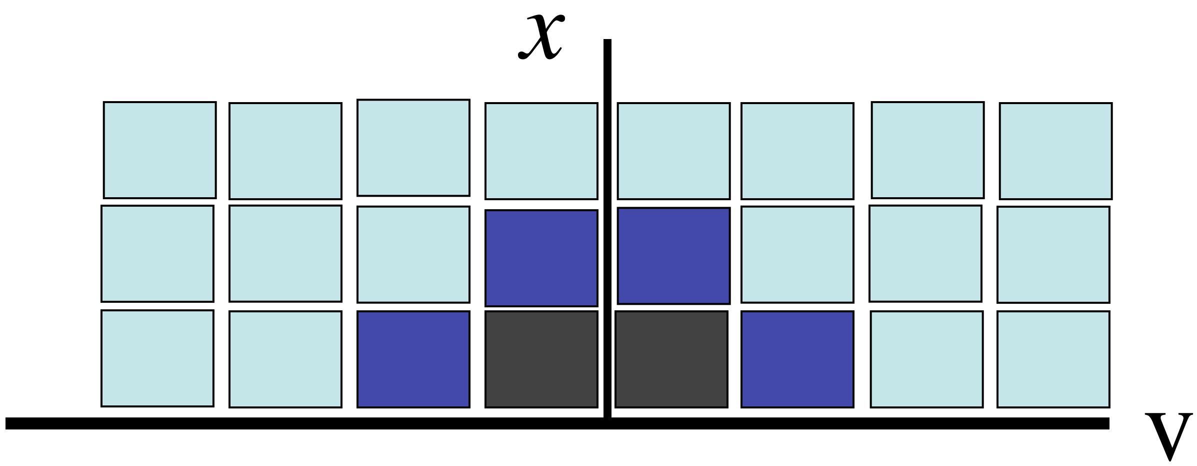
multiple steady state solutions

Rearrangement of Phase Space in Plasma

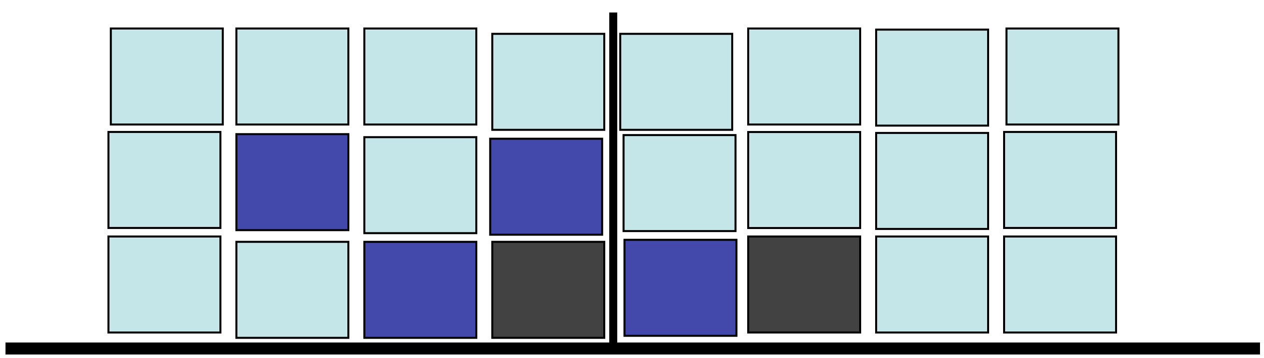


1. Current drive
2. One-way wall
3. Coupled diffusion in position-velocity: “alpha-channeling”

Gardener Restacking



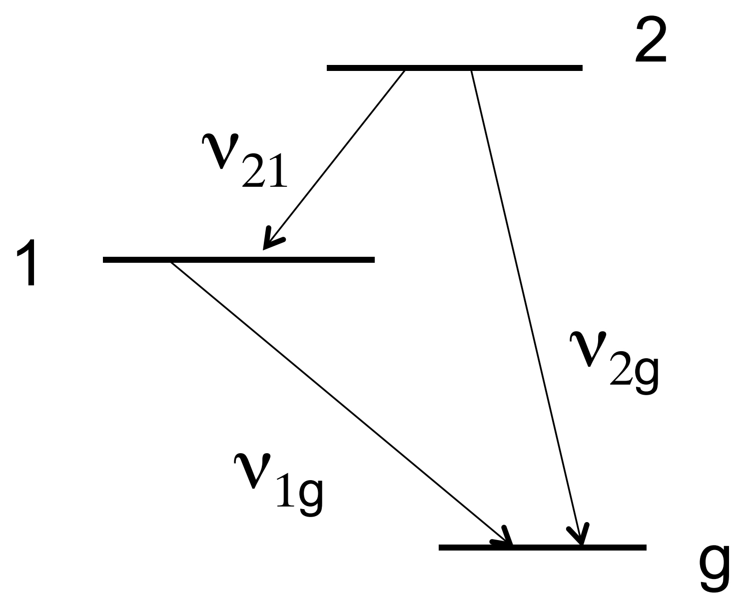
Minimum energy distribution



Initial energy distribution

Entropy conserving

Free Energy under Phase Space Rearrangement



$$\varepsilon = n_g \varepsilon_g + n_1 \varepsilon_1 + n_2 \varepsilon_2$$

minimized for: $n_g > n_1 > n_2$

more generally, minimize: $\varepsilon = \vec{n} \cdot \vec{\varepsilon}$

using π-pulse excitations ν_{ij}

Example: for $n_1 > n_2 > n_3$ t=0: $n_3\ n_2\ n_1$

To release free energy, apply 3 (ordered) π-pulses (to exchange densities)

ν_{21}: $n_3\ n_1\ n_2$ ν_{1g}: $n_1\ n_3\ n_2$ ν_{21}: $n_1\ n_2\ n_3$

Free Energy under *constrained* Phase Space Rearrangement

minimize: $\varepsilon = \vec{n} \cdot \vec{\varepsilon}$

under ***phase space conservation***

for atoms, use π-pulse excitations ν_{ij}: solution sequence $(\nu_{ij1}, \nu_{ij2}, \ldots)$

for plasma, use Hamiltonian forces: "Gardner restacking"

under ***diffusion constraint*** the free energy is not so easily found

for example: apply sequence (ν_{10}, ν_{21}) under diffusion constraint

	ε_0	ε_1	ε_2
Initial	n_0	n_1	n_2
Step1	$(n_1 + n_0)/2$	$(n_1 + n_0)/2$	n_2
Step2	$(n_1 + n_0)/2$	$(n_1 + n_0)/4 + n_2/2$	$(n_1 + n_0)/4 + n_2/2$

Fisch and Rax, 1993

Example

		$\varepsilon_0 = 0$	$\varepsilon_1 = 1$	$\varepsilon_2 = 4$
Initial	$W_0 = 22$	$n_0 = 0$	$n_1 = 2$	$n_2 = 5$
Step 1	$W_1 = 12$	5/2	2	5/2
Step 2	$W_2 = 45/4$	5/2	9/4	9/4

Apply (ν_{20},ν_{21})

		$\varepsilon_0 = 0$	$\varepsilon_1 = 1$	$\varepsilon_2 = 4$
Initial	$W_0 = 22$	0	2	5
Step 1	$W_1 = 21$	1	1	5
Step 2	$W_2 = 13$	3	1	3
Step 3	$W_3 = 10$	3	2	2

Apply $(\nu_{10},\nu_{20},\nu_{21})$

Better strategy

Strategy 1: Diffuse particles first between similar population levels

Example (continued)

		$\varepsilon_0 = 0$	$\varepsilon_1 = 1$	$\varepsilon_2 = 4$
Initial	$W_0 = 22$	0	2	5
Step 1	$W_1 = 35/2$	0	7/2	7/2
Step 2	$W_2 = 21/2$	7/4	7/2	7/4
Step 2	$W_3 = 77/8$	21/8	21/8	7/4

Apply $(\nu_{21}, \nu_{20}, \nu_{10})$

Best strategy

		$\varepsilon_0 = 0$	$\varepsilon_1 = 1$	$\varepsilon_2 = 4$
Step 1	$W_1 = 35/2$	0	7/2	7/2
Step 2	$W_2 = 63/4$	7/4	7/4	7/2
Step 3	$W_3 = 49/4$	21/8	7/4	21/8
Step 4	$W_4 = 175/16$	21/8	35/16	35/16

$(\nu_{21}, \nu_{10}, \nu_{20}, \nu_{21})$

Poor strategy

Strategy 2: Deplete particles first from high energy levels

Statement of the Problem

Discrete: Find the sequence $\{v_{ij}\}$ that minimizes: $W = \vec{n} \cdot \vec{\varepsilon}$

Continuous: Let

$$\frac{\partial f(v,t)}{\partial t} = \int K(v,v',t)\left[f(v',t) - f(v,t)\right]$$

$$K(v,v',t) = K(v',v,t)$$

$$K(v,v',t) \geq 0$$

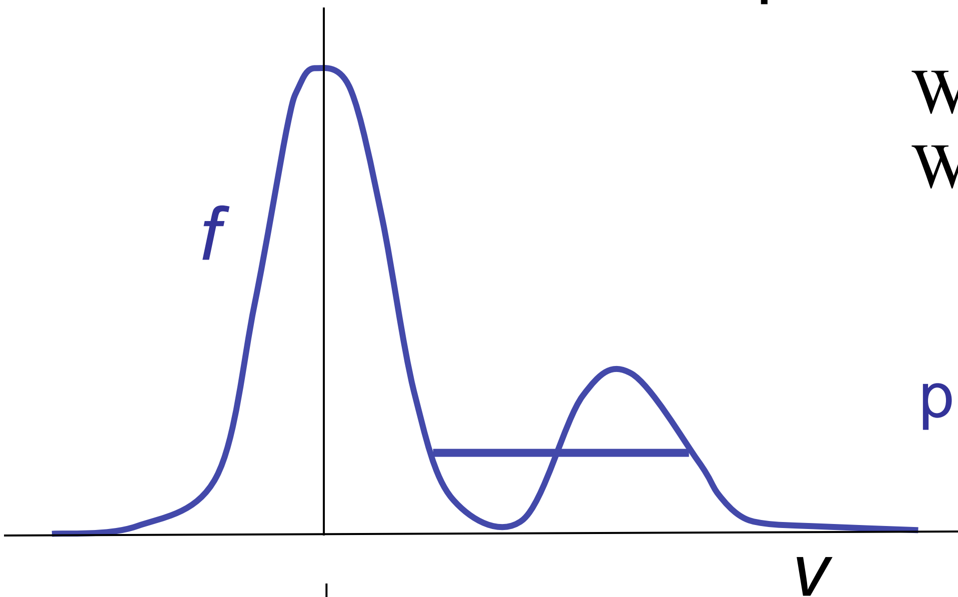
$$W(t) = \int \varepsilon(v) f(v,t)\,dv$$

Then find K that minimizes $W(t \to \infty)$.

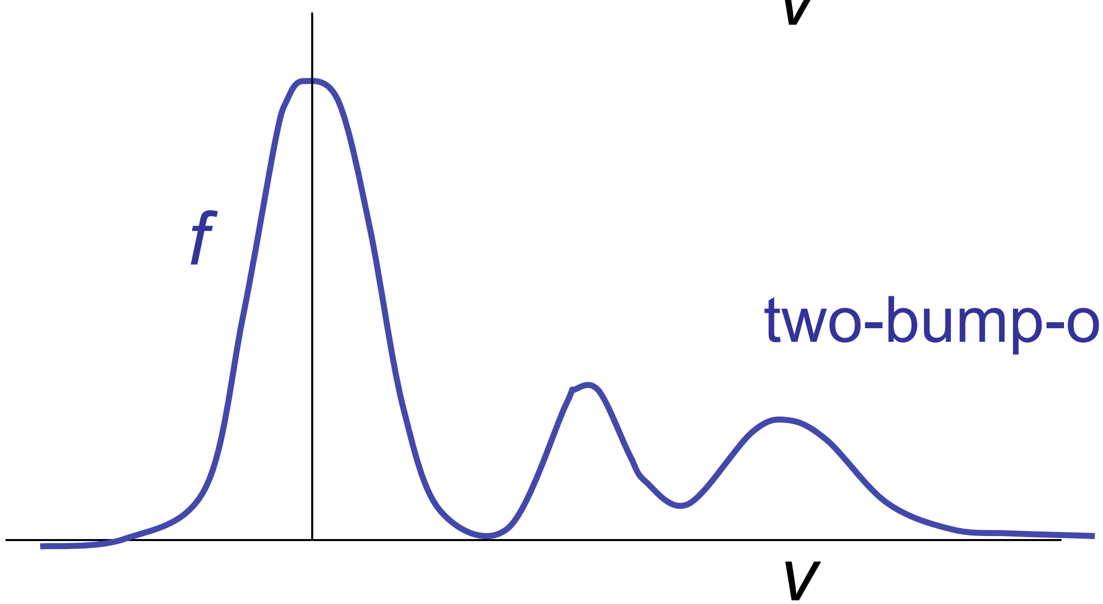
Note the H-theorem: $$\frac{d}{dt}\int f(v,t)^2 dv \leq 0$$

"Bump-on-tail" Instability

What is minimum energy state?
What is *complexity* of calculation?



plateau solution



two-bump-on-tail

multiple steady state solutions

Some Unsolved Challenges in RF Heating and Current Drive

1. Alpha Channeling – How to accomplish?

2. Current Drive effects associated with ions (optimize)

3. Neoclassical pinch effects and CD in ST

4. High-Efficiency Cyclic Operation
    1. What are the physical mechanisms for asymmetry
    2. Can Hyper-resistivity be induced?

5. Combine with Alpha Channeling
    1. What are the Engineering implications
    2. What are the associated physics issues?

6. Free Energy *Complexity Theory*