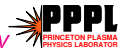


# Excitation of Alfvén modes by Energetic Particles in Magnetic Fusion

presented by N.N. Gorelenkov

*PPPL, Princeton University*

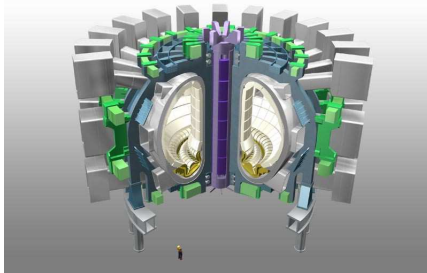


*Work supported in part by DoE contract No. DE-AC02-09CH11466*

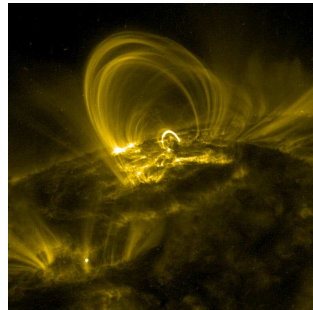
**Cadarache, France**  
**ITER, June 20, 2011**

## *Fusion is vital for mankind to survive*

controlled fusion on earth (ITER)



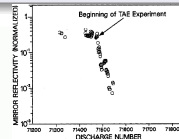
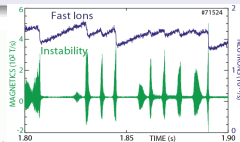
vs. therm. fusion in the sun



## Motivations to study energetic particle (EP) physics

- Heating for Deuterium-Tritium (DT) reactor: 1 GW DT plasma should have 200MW (20%) in fusion alpha particles.
- *Where does this power go ... e,i, lost?*
- *What are the mechanisms of power transfer?*
- *How to control it? Can we design an efficient fusion reactor?*
- *Can it damage the plasma facing components, diagnostics?*
- *Can we make use of EP instabilities for plasma diagnostics?...*

Lecture is far from systematic review, some names/labs are omitted, not consistent...



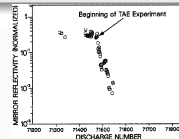
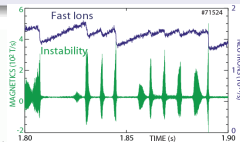
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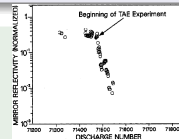
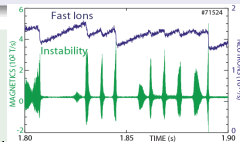
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  - "thermonuclear" instabilities
- 2 From single particle to collective effects
  - sources, parameters, distr. function...
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- 3 Alfvén (+other) waves, eigenmodes
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## Typical EP sources in present day (PD) experiments

Sources of energetic particles (ions):

- resonant frequency heating (RF):  $H$ ,  $50 \div 1000 \text{ keV}$ ,
- beam injection (NBI):  $D, H$ ,  $50 - 350 \text{ keV}$  ( $1 \text{ MeV}$  in ITER),
- fusion products:  $\alpha$  ( $3.52 \text{ MeV}$ ,  $DT$ ),  $p$  ( $3 \text{ MeV}$ ),  $T$  ( $1 \text{ MeV}$ ,  $DD$ )...
- test particles,

*EP, super-thermal ions:  $E_{EP} \gg T_{pi}$ ,  $v_{Ti} < v_{EP} \ll v_{Te}$*

*Relation to Alfvén velocity,  $v_A = B / \sqrt{4\pi\rho_i}$ , in fusion plasmas*

$$v_{Ti} = \sqrt{\beta_i} v_A < v_A \leq v_{EP}$$

### Corollary

- *EPs are decoupled from background ions for  $E_{EP} > 20 T_e$ , typically only electron drag (or RF) is present.*
- *characteristic drift, transit frequencies are much larger than those of thermal ions.*

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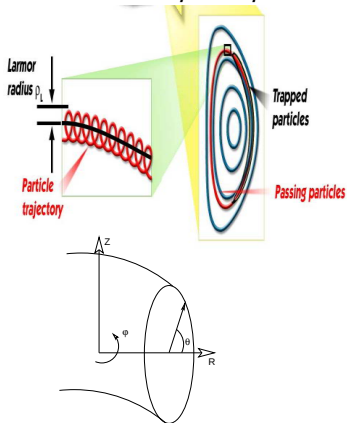
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## How EPs are confined? What is special?

Schematic of charged particle orbits in a tokamak  $\rho_{Li} \ll \rho_{Lf} \ll R$



### Plasma current estimate

- Larmor radius should be small enough  
(1)  $\rho_{Lf} \ll a$ ,  $\rho_{Lf} = v/\omega_c = vmc/eB_\phi$
- Poloidal Larmor radius (drift orbit width) determines confinement  $\Rightarrow$   
(2)  $a > \rho_{Lf} \theta = v_{\parallel} mc/eB_\theta$ .
- Find required plasma current via:  

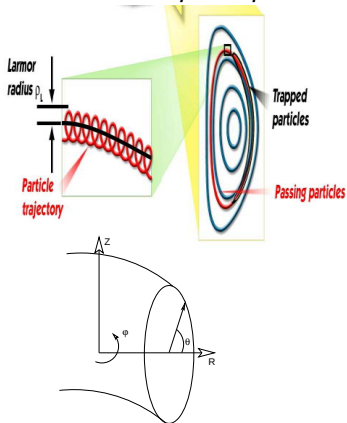
$$B_\theta [T] = \left(\frac{1}{2}\right) 0.2 I_{pl} [A] / a [cm]$$
 adiabatic moment conservation  

$$v_\perp^2 / B \simeq \text{const} \Rightarrow v_{\parallel} \simeq v \sqrt{2a/R} \Rightarrow$$

$$I_{pl/crit} [A] = \frac{10 \rho_{Li} [cm]}{\sqrt{R/2a}} B [Gauss] \simeq 2 \times 10^6$$
- In ITER plasma  $I_{pl} = 10 MA \gg I_{pl/crit}$ .
- *Ramp up regime can violate this.*

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## What is classical steady state EP distribution function

For drift-kinetic stationary equation,  $\partial f / \partial t = 0$

$$\frac{1}{\tau_{se} v^2} \frac{\partial}{\partial v} (v^3 + v_*^3) f + S(v) = 0$$

If  $S(v) = (4\pi)^{-1} S_0 \delta(v - v_0)$ , such as alpha's  $S_0 = \langle \sigma v \rangle n_D n_T$ :

$$\frac{1}{v^2} \frac{\partial}{\partial v} (v^3 + v_*^3) f + \frac{\delta(v - v_0)}{4\pi} S_0 \tau_{se} = 0,$$

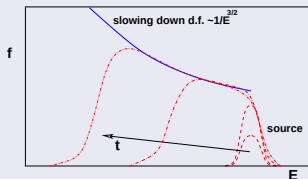
has a solution

$$f = \frac{S_0 \tau_{se}}{4\pi} \frac{v_0^2 H(v_0 - v)}{v^3 + v_*^3}.$$

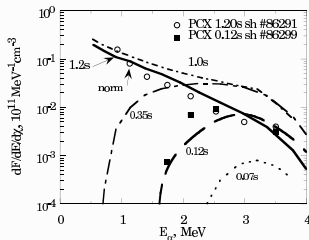
$H$  is  $\sim$  step function.

## Classical EP confinement is demonstrated in DT TFTR plasma

if plasma is not in steady state,  
i.e. beam like  
(fast start of the discharge)



TFTR *fusion* alphas

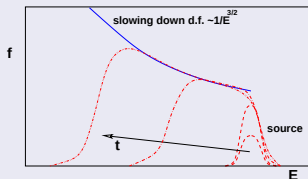


Numerical simulations of NPA in TFTR: more effects are included: (1) realistic geometry, (2)  $S = S(v, t)$  (Gorelenkov, NF'97)

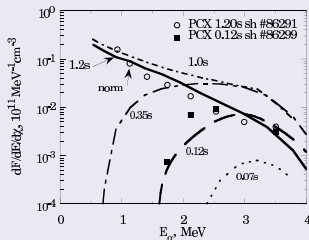
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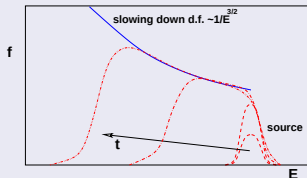


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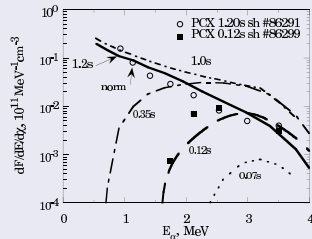
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*“New” development came from powerfull MHD codes*

such as NOVA, CASTOR > 1986 ...

*good place, good time ...*

MHD was well applicable for such “novel” problems:

- established codes, PEST, GATO, KINX, ...
- benchmarked against analytical solutions
- —//— other codes
- good problem for young researchers

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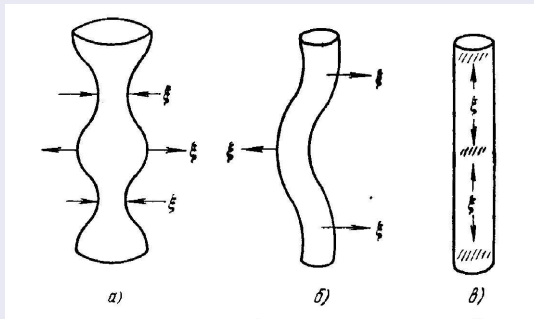
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## Three types of plasma oscillations can be of interest

compr. Alfvén

shear Alfvén

sound waves



Characteristic wave velocities:

$$\omega = k_{\parallel} v_A \text{ or}$$

$$\omega = k_{\parallel} v_s$$

## Shear Alfvén equation in nonuniform plasma

Use local Alfvén dispersion  $\omega^2/v_A^2 - k_{\parallel}^2 = 0$ .

Operate with

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$$

on equation for electrostatic potential or  $(div)(grad) : \left( \frac{\omega^2}{v_A^2} - k_{\parallel}^2 \right) \phi = 0$ .

We find shear Alfvén equation (AE)

$$\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\omega^2}{v_A^2} - k_{\parallel}^2 \right) r \frac{\partial}{\partial r} \phi + \left( \frac{\omega^2}{v_A^2} - k_{\parallel}^2 \right) \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \phi + Q \phi = 0.$$

Some other terms are included in  $Q$ , such as ballooning term.

Each radial point of  $\omega_A^2(r) = v_A^2 k_{\parallel}^2(r)$  corresponds to (AE) singularity and represents the continuum via  $v_A$  and  $k_{\parallel}$ .

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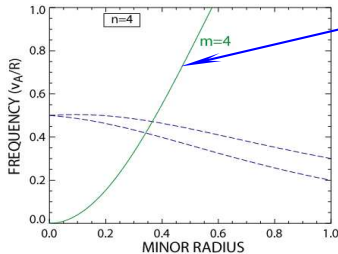
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*Direct local solution is damped*

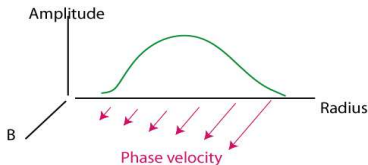


*local Alfvén continuum  $\omega_A(r)$ .*

Radially localized kinetic modes (KAW) are strongly damped

$$\gamma \sim d(k_{\parallel} v_A) / dr$$

Mechanism of the damping is through strong parallel electric field  $\Leftarrow$  thermal ion FLR + small scale.

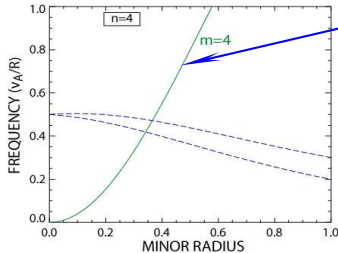


(Heidbrink, '08)

No much interest!

Rosenbluth '75, Mikhailovskii '75

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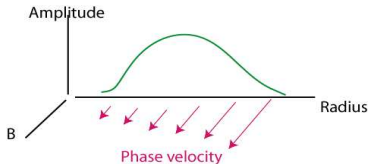


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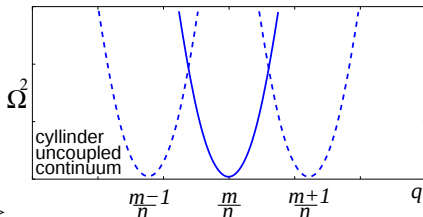
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## Shear Alfvén continuum schematic (cylinder, zero $\beta$ )

Magnetic field variation shapes the continuum through the safety factor profile  $q = B_\phi r / B_\theta R$ ,

$$\hat{k}_{\parallel} \phi = -i \frac{\vec{B}}{B} \cdot \vec{\nabla} \sum_m \phi_m e^{-i\omega t + im\theta - in\varphi} = \sum_m \left( \frac{m}{q(r)R} - \frac{n}{R} \right) \phi_m e^{\dots}$$



$$B = B(\theta, r), \quad \omega_A^2(r) = v_A^2 k_{\parallel}^2 \Rightarrow$$

Rational surfaces correspond to  $k_{\parallel m} = 0$ .

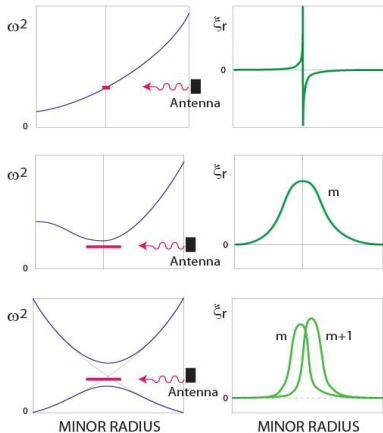
Solutions should account for the continuum structure.

If harmonics are coupled (nonhomogeneous) continuum gaps appear.

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## Various AE excitations are possible

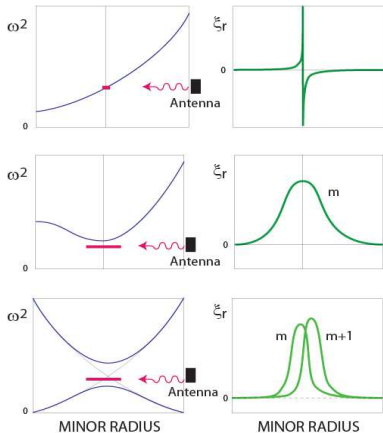


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- r-extended solution below local extremum (cylinder)
  - Solution is regular & spatially extended
- Gap creates conditions for broad mode

Natural instability can exist if damping is not high: (i) gap; (ii) near extremum (STs);

Pinches, PhD Thesis.

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Pinches, PhD Thesis.

## *Why continuum gaps are important?*

Gaps in the continua are common phenomena.

Example: energy spectrum gaps of valence electrons in a periodic potential well of a crystal lattice.

- Higher order effects are tested as coupling is described by small parameters: toroidicity  $\frac{a}{R}$ ; pressure  $\beta$ :... great test for theories,
- global modes may exist with frequencies inside gaps
- if modes with frequencies in the gaps exist they have low damping: no continuum damping from kinetic, singular mode.
- Can help to organize Alpha channeling (r-diffusion) together with the RF antennae (E-diffusion - N. Fisch talk)

## Realistic shear Alfvén continuum (schematic, torus)

Again take the eigenmode equation:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\omega^2}{v_A^2} - k_{\parallel}^2 \right) r \frac{\partial}{\partial r} \phi + \left( \frac{\omega^2}{v_A^2} - k_{\parallel}^2 \right) \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \phi + Q \phi = 0.$$

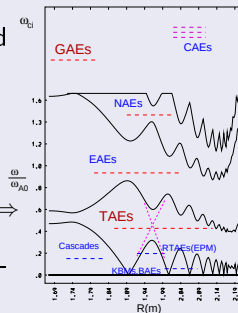
If pol-/toroid. harmonic expansion is applied

$$\phi = \sum_m \phi_m \exp[-i\omega t + im\theta - in\varphi],$$

geometrical effects couple harmonics:

$$v_A^2 k_{\parallel}^2 = B^2(r, \theta) k_{\parallel}^2 / 4\pi\rho; B = B_0(1 + \varepsilon \cos\theta \dots) \Rightarrow$$

(high order) toroidicity, ellipticity, triangularity, ..., pressure induced gaps.



(Cheng, Chance '86, K.L. Wong '91, W. Heidbrink '91)

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## General procedure for ideal MHD codes

Based on m-truncated expansion

$$\phi = \sum_{m,l}^{M,L} \phi_{m,l} U_l(\psi) \exp[-i\omega t + im\theta - in\varphi],$$

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\omega^2}{v_A^2} - k_{\parallel}^2 \right) r \frac{\partial}{\partial r} \begin{pmatrix} \dots \\ \phi_m \\ \phi_{m+1} \\ \dots \end{pmatrix} + \left( \frac{\omega^2}{v_A^2} - k_{\parallel}^2 \right) \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \begin{pmatrix} \dots \\ \phi_m \\ \phi_{m+1} \\ \dots \end{pmatrix} + \\ + Q \begin{pmatrix} \dots \\ \phi_m \\ \phi_{m+1} \\ \dots \end{pmatrix} = 0 \Rightarrow \end{aligned}$$

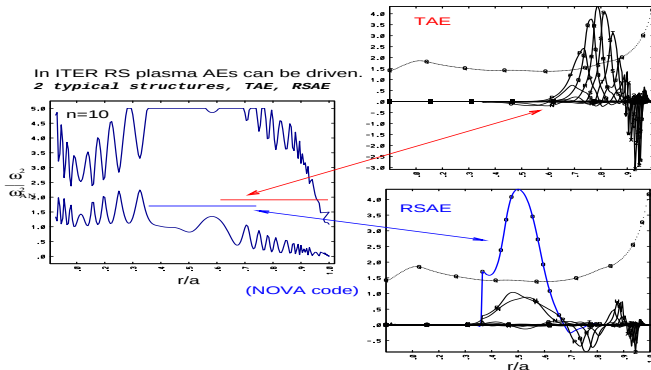
Sophisticated geometrical effects are present.

Non MHD effects can be present perturbatively or nonperturbatively:

FLR, kinetic ...

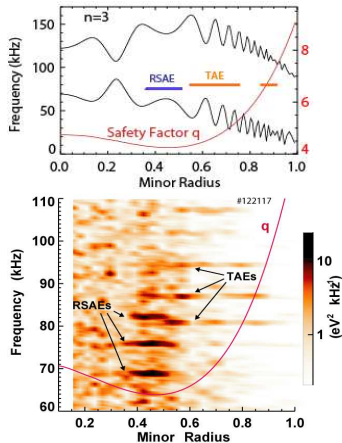
Analytic theories employ sometimes similar approach.

*Generally TAE structure is complicated in realistic geometry*



RSAs and TAEs can be unstable, lead to  $\alpha$ , beam losses in ITER.  
 Both modes are known to induce beam losses in DIII-D.

## A remarkable agreement between TAE, RSAE mode structure in DIII-D and NOVA modeling (ideal MHD)

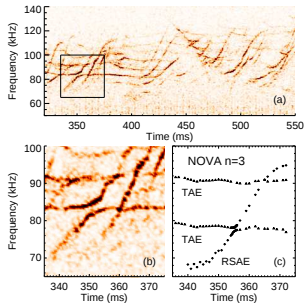


- NBI into  $\sim$ low density DIII-D plasma
- Various gap modes reside in an effective waveguides, such as caused by  $q_{min}$  presence
- RSAEs are localized near continuum extrema points
  - RSAEs have one dominant poloidal harmonic
- Both RSAE and TAE structures agree quantitatively with ideal MHD (NOVA simulations)

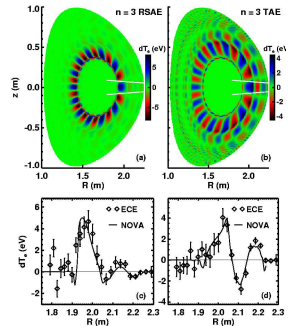
## Ideal MHD TAE structure agrees with ECE

Internal TAE/RSAs mode structures measured by ECE show excellent agreement with ideal MHD predictions (*NOVA calculations, Van Zeeland, PRL '06*)

### RSAs & TAE frequencies crossover



### RSAs & TAE structures



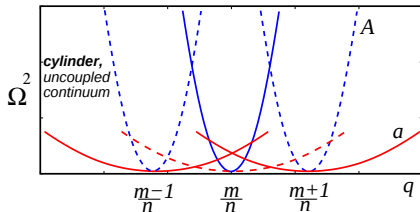
It could be that earlier works (*Carolipio '01, Heidbrink '97*) dealt with other modes such as EPs in high fast ion beta plasma.

# Outline

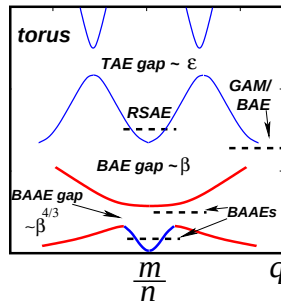
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## Plasma pressure introduces BAAE (Alfvén-acoustic) gaps/modes

- Alfvén (A) continuum at low frequency:  $\Omega^2 = v_A^2 k_{0,\pm 1}^2 / (1 + 2q^2)$  (modified)
- Acoustic (a) branch  $\Omega^2 = [\gamma\beta v_A^2 / 2(1 + \delta)] k_{0,\pm 1}^2$  is coupled via  $m \pm 1$  sidebands with modified Alfvén continuum ( $m$  harmonic) due to geodesic curvature and pressure.



$\Rightarrow$  gaps



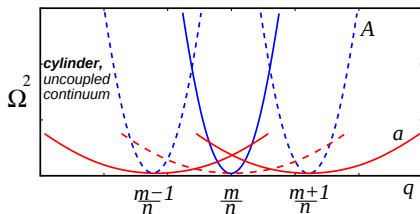
**BAAE gaps are due to  $\beta$  and geodesic curvature effects**

Global modes exist in A-a gaps

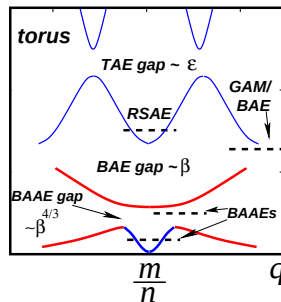
Chu'92, Van der Holst'01, Gorelenkov'07

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**BAAE gaps are due to  $\beta$  and geodesic curvature effects**

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Chu'92, Van der Holst'01, Gorelenkov'07

## Alfvén/acoustic continuum bounds global modes

Shear Alfvén and acoustic continua equations capture main effects  
in low- $\beta$ , large aspect ratio plasma, low  $\omega_*$ , (Cheng, Chance, PFI '86):

$$\Omega^2 y + \partial_{\parallel}^2 y + \gamma \beta \sin \theta z = 0 \text{ (Alfvénic)} \quad (1)$$

$$\Omega^2 \left(1 + \frac{\gamma \beta}{2}\right) z + \frac{\gamma \beta}{2} \partial_{\parallel}^2 z + 2\Omega^2 \sin \theta y = 0 \text{ (acoustic)}, \quad (2)$$

where  $\Omega \equiv \omega R / v_A$ ,  $y \equiv \xi_s \varepsilon / q$ ,  $\xi_s \equiv \vec{\xi} \cdot \frac{[\mathbf{B} \times \nabla \psi]}{|\nabla \psi|^2}$  and  $z \equiv \nabla \cdot \vec{\xi}$ ,  $\hat{k}_{\parallel} \equiv i \partial_{\parallel} / R$ .

Geodesic curvature coupling:  $m$  Alfvénic and  $m \pm 1$  acoustic harmonics.

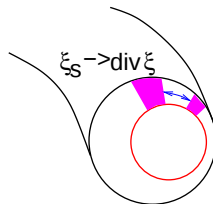
### Various solutions exist

uncoupled **acoustic (a)**  $\Omega^2 = \frac{1}{2} \gamma \beta k_{\parallel}^2 R^2$

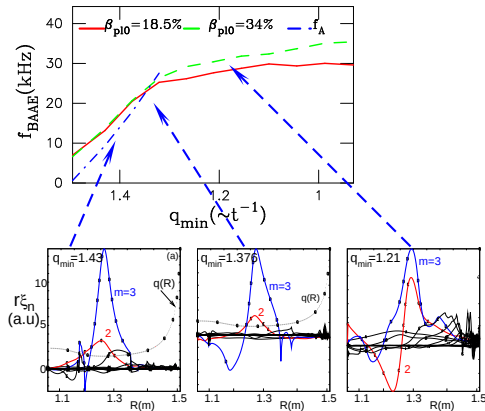
and **Alfvénic (A)** branches  $\Omega^2 = k_{\parallel}^2 R^2 + \Omega_{GAM}^2$ .

**GAMs:**  $\Omega_{GAM}^2 = \gamma \beta (1 + 1/2 q^2)$

**modified shear Alfvén** branch  $\Omega^2 = \frac{k_{\parallel}^2 R^2}{1 + 2q^2}$



## NOVA: gap BAAE broadens as $q_{min}$ decreases in NSTX

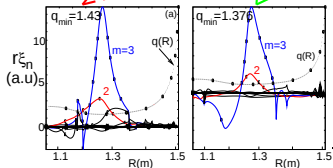
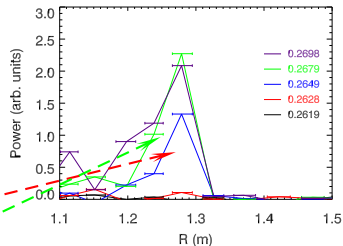
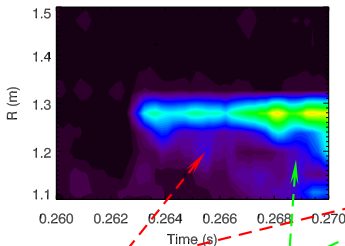


- BAAE frequency sweeps as  $q$ -profile relaxes.
  - $f$  does not depend on beta near rational  $q_{min} = 1.5$ .
  - $\xi_r$  has one dominant harmonic  $m = nq_{min} = 3$ .
- $f_{BAAE}$  is close to modified Alfvén branch  $f_A = v_A k_{||} / 2\pi \sqrt{1 + 2q_{min}^2}$
- Continuously transforms to broad gap mod
- Gaps shape BAAEs

# Ultra SXR measures the same radial structure broadening

Raw USXR signal ( $\sim$ BAAE structure, *Tritz, JHU*)

Radial profile evolution



BAAE broadens as  $q_{min}$  decreases  
Other tokamaks observed BAAEs  
JET, DIII-D, ASDEX ...

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## EP driven instabilities

Three conditions should be satisfied:

- source of free energy must be present
- particles should be at resonance
- drive should be *higher than the damping*

Free energy sources:

- pressure radial gradient (universal drive), low frequency
- velocity space gradient, high frequency
  - inversed velocity distribution
  - velocity anisotropy

*As an example we construct the universal instability drive*

$$\frac{\gamma}{\omega} = K^{-1} \left[ \Delta \mathcal{E} \beta_{\alpha} \frac{\partial f(v_{res})}{\partial \mathcal{E}_{\alpha}} + \frac{\Delta r}{r} \frac{\partial \beta_{\alpha}}{\partial \ln r} f(v_{res}) \right] \sim -\omega \beta_{\alpha} f(v_{res}) + \frac{n}{\omega_c} \frac{\partial \beta_{\alpha}}{\partial \ln r} f(v_{res}) \sim n.$$

*Slow ramp up of the alpha source  $\Rightarrow$  avoid thermonucl. instability*

## Most unstable mode number scales with the machine size

Among possible Alfvén eigenmodes toroidicity-induced AE (TAEs) modes are likely to **limit fusion product confinement in BP**.

Low frequency modes are mostly responsible for fast ion radial transport: TAEs, EPMs, BAEs.

High frequency modes are responsible for phase space particle diffusion: GAEs, CAEs, ICE.

Theory:

$n$  range of most unstable TAEs determined by Finite Orbit Width (FOW) effects: (Berk PL, '92, Fu PF, '92, Breizman '95, Candy '95)

$$k_{\perp} \Delta_b \simeq \frac{n q^2 \rho_b}{r} \sim 1 \Rightarrow \frac{r}{R} n_{\max} < n < n_{\max} \simeq \frac{r \omega_{c\alpha}}{q^2 v_A}$$

Codes (NOVA) show agreement with theory

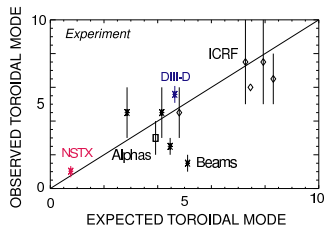
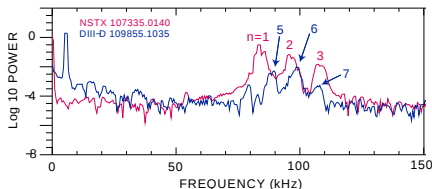
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## Experimental observations confirm theory predictions

DIII-D/NSTX similarity experiments were designed to confirm theory predictions (*W. Heidbrink, PPCF '03*):

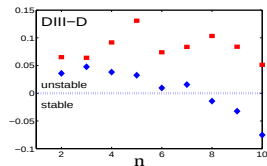
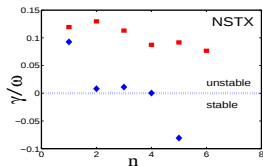
- The  $a = 0.8m$  radius in NSTX and DIII-D but different  $R = 1m$  vs  $R = 1.7m$ ,  $B = 0.6T$ .
- Use similar NBI features: injection geometry, energy, trapped to passing particle ratio.



Most unstable mode number scales as

$$n \sim a/q^2$$

## *Trend of unstable TAE's $n$ -dependence is recovered by theory*



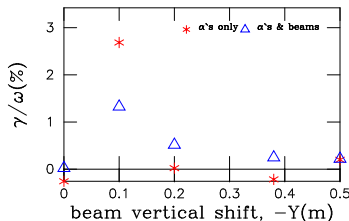
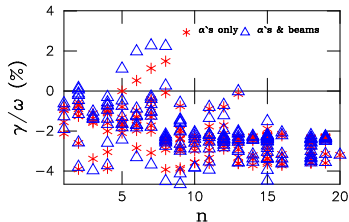
■ no damping, ◆ - with damping

- TAEs excitation thresholds are reproduced for medium- $n$  numbers
- NOVA-K with isotropic d.f. does not predict observed unstable modes
- Main damping mechanisms are:
  - ion Landau damping (dominant in ITER), radiative damping
- Stabilization of TAEs at high end of  $n$  range is due to FOW effects and higher damping
- **Helps to validate predictive capabilities of theory.**

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## Numerical simulations of AE stability in ITER

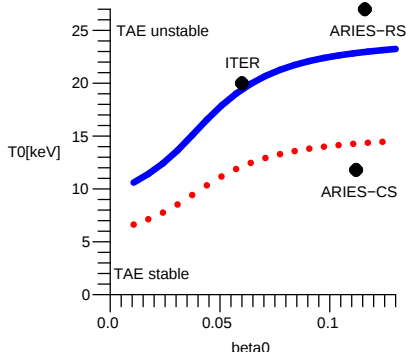


1 MeV beam ion drive is comparable with the  $\alpha$ -drive.

- lowering NBI energy to 0.5 MeV significantly reduces drive.
- medium, high- $n$ 's are unstable  $n = 6 - 13$ .
- most unstable modes are localized at  $r/a \sim 0.5$  and avoid central ion Landau damping.
- TAE control is possible via NBI aiming.

## Projections to ITER and other BPs designs

Analytical model for TAE stability in BPs with drive from beams and alphas



- Stability diagram in  $\beta_{pl}(0) / T_i(0)$ 
  - source of alphas comes from background plasma  $S_\alpha \sim \langle \sigma v \rangle \Rightarrow \frac{-\partial \beta_\alpha}{\partial \ln r} \simeq \frac{7}{2} \beta_\alpha \frac{-\partial \ln T}{\partial \ln r}$
  - theory (dashed curve), normalized to NOVA (solid curve)
- $\alpha$ 's slowing down d.f., ion Landau and trapped electron collisional dampings.
- see *K.Ghantous poster for details.*

## Linear theory

- well developed, routinely finds zoo of modes
- theory is validated
- problems exist: kinetic effects, damping mechanisms are not tested, comprehensive codes are built.

## Non-linear theory

- Nonlinear/transport with multiple instabilities should be #1 priority:
  - Develop realistic reduced models
  - Realistic, global nonlinear codes
  - V&V needed (ITPA?)
- Experiments on the “sea of Alfvén modes” driven transport should be done on present day machines.

## *Effect of instabilities on energetic ions depends on frequency*

If cyclotron interaction is ignored we have

$$\frac{\Delta P_\phi}{n_\phi} = \frac{\Delta \mathcal{E}_{EP}}{\omega}$$

or

$$\frac{\Delta r}{r} \frac{\mathcal{E}_{EP}}{\Delta \mathcal{E}_{EP}} = \left( \frac{nq}{r} \rho_{EP} \right) \left( \frac{\rho_{EP}}{2r} \frac{\omega_c}{\omega} \right).$$

- Low frequency instabilities ( $< 200 kHz$ ) may induce radial particle transport
- High frequency instabilities affect velocity space transport.